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A STUDY OF THE COST-EFFECTIVENESS OF INVENTORY MANAGEMENT POLIC--ETC(U)

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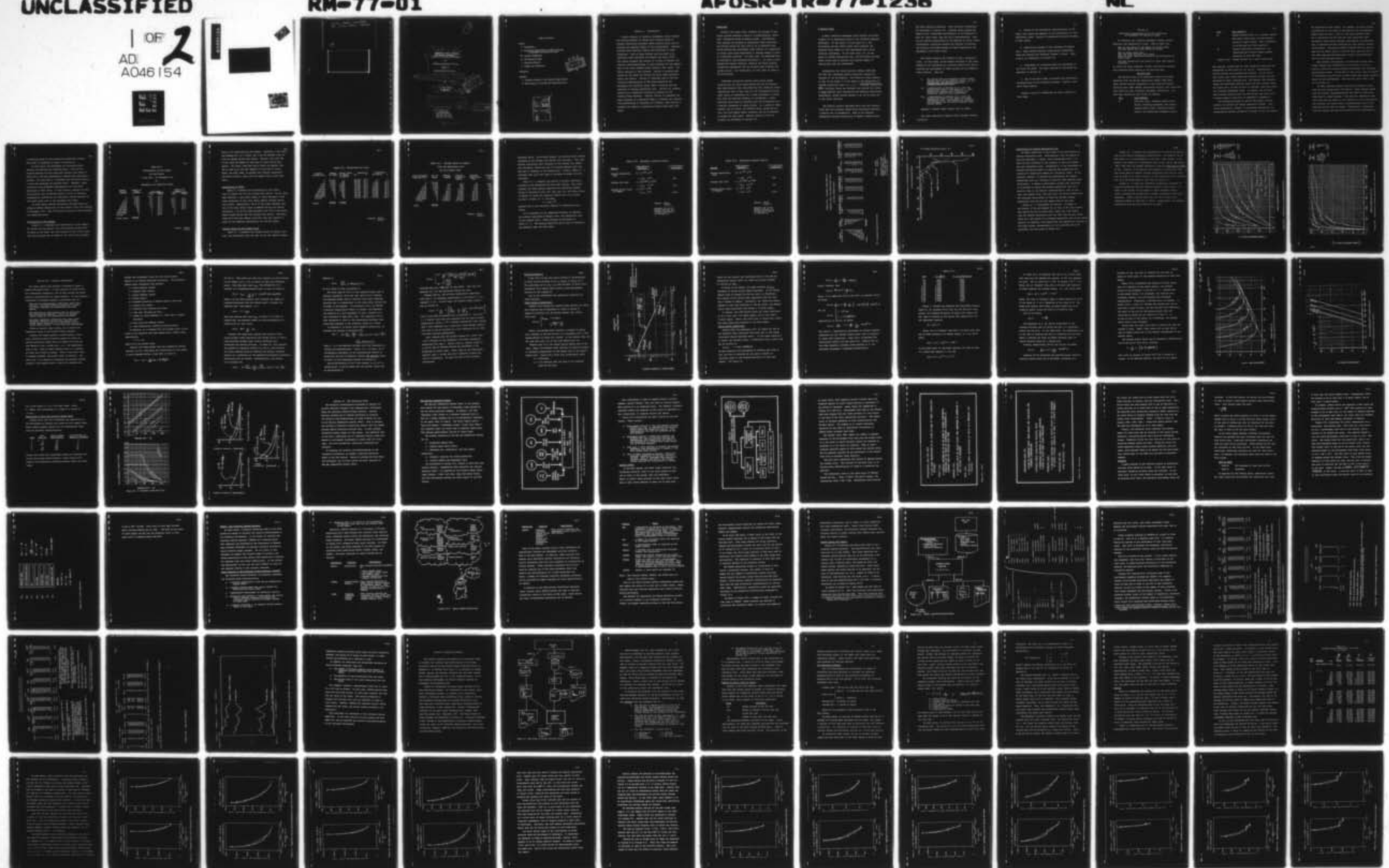

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6 A STUDY OF THE COST-EFFECTIVENESS

OF

INVENTORY MANAGEMENT POLICIES

BASED ON

AVERAGE REQUISITION SIZE.

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by

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Section I. Introduction

A major question of inventory management policy concerns the relative benefit of safety level formulas based on average requisition shortages versus formulas which seek to minimize the expected number of units backordered. Appendix A outlines three specific formula sets addressed to these viewpoints. Each of these formula sets are based on an analytical model developed by Presutti and Trepp (1970). This report presents the results of a study to estimate the relative cost-effectiveness of these formulas for the management of Air Force Economic Order Quantity (EOQ) items. The report consists of five sections. Section I provides background for the study and reviews the major tasks performed during this effort. Section II discusses some of the statistical characteristics of EOQ items managed by the Sacramento Air Logistics Center (ALC), including the distribution of average requisition size. Section III presents results of an analytical sensitivity analysis of the Presutti-Trepp formulas. Finally, Section IV discusses the major features of a simulation model to evaluate the relative cost-effectivness of alternate P-T formulas, while Section V presents the results of simulation studies based upon this model.

Background

Presutti and Trepp (1970) consider the problem of computing optimum inventory levels in a single-echelon, multi-item, continuous review inventory system. Specifically, they consider the problem of determining order quantities and reorder points for each item so as to minimize total system holding and procurement costs subject to a constraint on either total units backordered or average number of units in a backorder position. In either case, the backorders may be weighted by item essentiality-factor Z. By using a novel probability density function, Presutti and Trepp obtained closed-form expressions for the optimum order quantity and reorder points. For convenience, we will refer to these as the PT-formulas.

Subsequent simulation studies using actual demand histories for Air Force items showed that the PT-formulas were significantly more cost-effective than inventory levels computations then in use; that is, the PT-formulas provided lower levels of backorders for a given investment in inventory than the previous formulas, or conversely, a given backorder level could be achieved with the PT-formulas with a smaller investment in safety stocks. As a result of these studies, the Air Force, the Defense Supply Agency and the Army (for high demand items) currently use the PT-formulas to manage EOQ type items. Specific details of the PT-formulas are discussed in Section III.

A Current Issue

A major inventory management issue concerns the establishment of an appropriate factor Z for weighting expected backorders. If the factor Z is set equal to 1, the PT-formulas provide safety stocks which minimize the expected total number of units backordered for a given inventory investment. On the other hand, if Z is set equal to average requisition size, the PT-formulas provide levels which tend to minimize the expected number of requisitions that are backordered.

Headquarters Air Force Logistics Command (AFLC/LOR and XRS) has performed several analytical sensitivity analyses of the PT-formulas. One finding of these studies is that if the factor Z is set equal to the square-root of average requisition size, i.e., if $Z = \sqrt{\text{Average Requisition Size}}$, inventory levels are produced that provide low levels of <u>both</u> expected units backordered and expected requisitions backordered, though neither measure is as low as under one of the "pure" policies.

The analytic results described above are only strictly valid when the inventory system being considered satisfies a specific set of assumptions. Some of the required assumptions include stationarity of demand, stable prices,

and known delivery leadtimes. Other important assumptions are discussed in Section III. Although these assumptions appear to be a reasonable approximation of the behavior of many Air Force items, they are still approximations. They are not precise descriptions of the behavior of actual items. Consequently, simulation studies are required to evaluate the relative cost-effectiveness of these formulas for the management of Air Force items.

This report presents the results of such a simulation study. In this study, actual demand histories of EOQ items managed by the Sacramento Air Logistics Center were used to simulate the performance of three versions of the Presutti-Trepp formulas. They are:

I. The current EOQ Buy Computation System (D062) reorder level and buy quantity rules. In this alternative, the essentiality factor Z = $\sqrt{\text{Average Requisition Size}}$.

II. A modification of formula set I, with the essentiality factor Z set equal to 1. This formula set contains the PT-formula for minimizing expected units backordered.

III. A modification of formula set I, with the essentiality factor Z set equal to average requisition size. This formula set contains the PT-formula for minimizing expected requisitions short.

Appendix A defines these formula sets in detail.

The study consisted of several major research efforts, including:

a. Studies of the statistical characteristics of EOQ items, with particular emphasis on the distribution of item requisition sizes. Section II summarizes the major findings of this effort.

b. Sensitivity analyses of the alternate PT-formula sets. These studies identify computational differences among the formulas for different classes of items. These results are summarized in Section III.

c. Development of a detailed model for simulation of Air Force EOQ items. The major features of this model are described in Section IV.

d. Use of the above model to estimate the performance characteristics of the alternate formulas. Section V presents these results.

Finally, Section VI summarizes the major findings of this study.

Section II.

Statistical Characteristics of EOQ Inventories at Sacramento Air Logistics Center

In analyzing any inventory management system, several questions come immediately to mind. Some of these are:

> What is the nature of the demand for these items?
> How does demand vary from period to period?
>
> What do these items cost?
> What are their delivery lead times?
> What are other significant physical characteristics of these items?
>
> How does requisition size relate to other item characteristics?

To answer the above questions, several statistical analyses were conducted using records in the EOQ data bank.

EOQ Data Bank

The EOQ data bank is an historical archive of records extracted from the EOQ Buy Computation System (AFLC D062). These records contain information on Air Force and Foreign Military Sales (FMS) demand, serviceable returns, unit prices and lead times and other inventory management information. The data bank consists of four files, as follows:

File	Data Elements
01	Item unit price, inventory status information, inventory management and program identification codes, inventory control levels, and requisition frequency counts.

File	Data Elements
02	Demand activity data; i.e. transfer demand, FMS sales, other sales, and serviceable returns to depot stocks. This information is by quarters for eight quarters.
03	Program application information; i.e. identification of major weapon systems supported by each EOQ item.
History File	Demand history for a given fiscal year.

Each quarter, records from the D062 system are extracted and reformatted to form files 01, 02, and 03. Approximately 500,000 records are processed each quarter. At the end of each fiscal year, data tapes from previous quarters are consolidated to form a History File. The History File contains demand and serviceable returns counts for each quarter within the fiscal year, as well as data on leadtime, item unit costs, and inventory management codes. At present, the EOQ data bank contains History Files for each fiscal year since 1971. A record layout of this file is presented in Appendix C.

We originally planned to use the EOQ history records directly to simulate the demand generation process. Upon closer examination, however, we discovered substantial inconsistencies between information recorded in the unit demand

and requisition count fields. For example, for many records a requisition of one might be recorded but the corresponding demand field was zero. That is, the data indicated there was a requisition for NO units for that particular item. Conversely, many records recorded some demand, but indicated that no requisitions were submitted for that period, another inconsistency. We examined several hundred items in various ALCs for the fiscal years of 1975 and 1971, and over 30% of the items examined contained major logical inconsistencies such as those mentioned above.

Upon further investigation, we found that several data system changes were implemented in the spring of 1976 to correct timing and accuracy problems associated with recording requisitions counts. At the time of this study, only one period of information was available in which these data system improvements had been implemented. This was the period from 1 July 76 to 31 Sept 76.

We then conducted several statistical analyses of data from the July through Sept 1976 period. These studies showed that the logical inconsistencies we had found in older data files were no longer present. This does not necessarily mean that there were no errors in the recording of this information, but certainly the gross logical inconsistencies that we had detected earlier were not found. Consequently, information from this period was used as the basis for our development of

a simulation model of the requisition generation process. This model is discussed in detail in Section IV.

At this point, the Sacramento Air Logistics Center (SM-ALC) was selected for detailed study. This ALC was selected because its data files were smaller than those of other ALC's, and thus represented a smaller data processing problem. Two other ALC's -- Oklahoma City and Warner Robins -- were also selected for detailed study. Together, these three ALC's are considered representative of the entire population of AFLC items. At this writing, analysis of data from Oklahoma City and Warner Robins is still in progress. Consequently, statistical and simulation results reported in this paper apply only to the Sacramento ALC items.

As noted above, several statistical analyses were performed of SM-ALC demand data for the period 1 July 1976 through 31 September 1976. Some of the major results of these analyses are summarized below.

Distribution of Unit Demand

Table II - 1 describes the distribution of unit demand for SM-ALC for the period 1 July 1976 through 30 Sept 1976. As shown in the table, the vast majority of the 78,436 items that were analyzed had no demand at all during this quarter,

Table II-1

Distribution of Unit Demand

For the Period

1 July 1976 - 30 September 1976

At

Sacramento Air Logistics Center

Demand (Units)	Number of Items	All Items		Items with Demand 0	
		%	Cum %	%	Cum %
0	66,229	84.4	84.5		
1	3,313	4.2	88.7	27.1	27.1
2-3	2,893	3.7	92.2	23.7	50.8
4-10	3,130	4.0	96.2	25.6	76.4
11-31	1,717	2.2	98.5	14.1	90.5
32-100	819	1.0	99.5	6.7	97.2
101-316	245	.3	99.8	2.0	99.2
317-1000	67	.1	99.9	.5	99.7
1001-3162	16	-	99.9	.1	
3162-10000	4	-	99.9	.0	
10,000+	3	-	100.0	.0	
Total Items	78,436				

Source: N084T
3/5/77

while 3,313 items had only one demand. Similarly, 2,893 items had demands for 2 or 3 units, and 3,130 had between 4 and 10 units of demand during this period. Overall, more than 96% of the items had demand of less than 31 units during this period. For those items that had at least one demand, over 90% of them still had unit demands of less than 31 units. Hence, for most items, it appears that average requisition size must be small, since the unit demand rate is also very small.

Distribution By Value

Table II - presents the distribution of the dollar value of demand for the July through Sept period. As has often been observed, a very small number of items represent a very large proportion of the total dollar demand recorded during this period. Specifically, 98.5% of the items represent just 64% of the total dollar value of demands during this period. Conversely, 1.5% of the items represent over 36% of the dollar demand placed during the July through Sept period. Obviously, items with high unit demand rates have very high impacts in terms of the budgetary requirements for the EOQ system.

Average Values By Unit Demand Class

Table II - 3 presents the average values of demand, unit cost, and requisition size for each of the unit demand classes

Table II-2 Distribution by Value

Demand Jul-Sep 1976	Number of Items	Dollar Value Of Demand (1,000's)	Cumulative No.	Cumulative %	Cumulative $(1,000's)	Cumulative %
0	66,229	-	66,229	84.5	-	-
1	3,313	937	69,542	88.7	937	6.7
2-3	2,819	1,395	72,361	92.2	2,332	16.7
4-10	3,130	2,892	75,491	96.2	5,225	37.6
11-31	1,717	3,657	77,208	98.5	8,882	63.9
32-100	819	1,978	78,027	99.5	10,861	78.2
101-316	245	2,093	78,272	99.9	12,954	93.2
317-1000	67	514	78,339	99.9	13,469	97.0
1001-3162	16	211	78,355	99.9	13,680	99.5
3163-10000	4	120	78,359	99.9	13,800	99.4
10,000+	3	80	78,362	100.0	13,881	100.0
Totals	78,436	13,881				

Source: G570T
4/16/77

Table II-3. Average Values of Demand,

Cost and Requisition Size

By Unit Demand Class

Units Demanded July-Sep 1976	No. of Items	Average Demand	Average $-Value of Demand	Average Requisition Size	Average Unit Cost
0	66,229				
1	3,313	1.0	283.	.98	$283
2-3	2,819	2.3	495.	1.64	211
4-10	3,130	6.1	924.	3.16	155
11-31	1,717	17.8	2,130.	6.19	118
32-100	819	54.5	2,416.	13.94	46
101-316	245	164.1	8,545.	28.48	59
317-1000	67	507.5	7,681.	97.25	14
1001-3162	16	1,952.0	13,204.	120.31	9
3163-10000	4	6,384.0	30,010.	97.25	4
10,000+	3	26,929.0	115,829.	189.00	7
Total	78,436				

Source: G570T
4/16/77

discussed above. As we would suspect, the average dollar demand increases as the average unit demand rate increases. Also, the average requisition size increases as the average unit demand rate increases, though the rate of increase appears less than the rate of increase in unit demand rate. Finally, Table II - 3 indicates that unit cost tends to decrease as demand activity increases.

Table II - 4 presents the results of several regression analyses of the Sacramento EOQ Data Bank records. This table presents the least squares equations for estimating average values of requisition size (R), unit cost (C), and dollar value of demand (V) given the quarterly demand rate (D) in units. As shown in Table II - 4, the model

$$R = .925\ D^{.54}$$

explains 58% of the variation observed in requisition size values.

As a by-product of our regression studies, we computed the average logarithms of demand, cost, and requisition size by unit demand class. These averages are displayed in Table II - 5. The average logarithms may be used to determine the geometric mean for each class.

Table II-4. Regression Analysis Results

Measure	Least-Squares Equation	% of Variation Explained
Average Requisition Size	$R = e^{-.077} D^{.54}$	
	$= .925 \quad D^{.54}$	58%
Average Unit Cost	$c = e^{4.3099} D^{-.4568}$	
	$= 74.4 \quad D^{-.46}$	10%
Average Dollar Value of Demand	$V = e^{4.3} D^{.54}$	
	$= 73.6 \quad D^{.54}$	13.6%

Source: G570T
4/16/77

Analysis of 12,207 SM-ALC records, all records with DEM > 0 in Jul-Sep 76.

Table II-4. Regression Analysis Results

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	$= 74.4\ D^{-.46}$	10%
Average Dollar Value of Demand	$V = e^{4.3}\ D^{.54}$	
	$= 73.6\ D^{.54}$	13.6%

Source: G570T
4/16/77

Analysis of 12,207 SM-ALC records, all records with DEM > 0 in Jul-Sep 76.

Table II-5. Average Logarithms of

Demand, Cost, and Requisition Size

By Unit Demand Class

Demand Jul-Sep 1976	Number of Items	ln [Demand (Unit)]		ln [Demand ($)]		ln [$\bar{R}$]*		ln [Unit Cost($)]	
		u	σ	u	σ	u	σ	u	σ
0	66,229	-	-	-	-	-	-	-	-
1	3,313	-	-	4.26	1.83	-.06	.24	4.26	1.83
2-3	2,893	.83	.19	4.78	1.89	.35	.45	3.94	1.88
4-10	3.130	1.77	.30	5.30	1.91	.89	.70	3.53	1.90
11-31	1,717	2.83	.30	5.95	1.93	1.48	.87	3.11	1.92
32-100	819	3.94	.32	6.36	1.85	2.07	1.05	2.41	1.87
100-316	245	5.05	.33	6.91	1.90	2.62	1.16	1.86	1.90
317-1000	67	6.18	.30	7.15	1.87	3.32	1.59	.97	1.87
1001-3162	16	7.53	.33	8.16	1.91	4.40	.92	.63	2.02
3163-10000	4	8.71	.39	8.49	2.39	4.49	.52	-.21	2.12
10,000+	3	10.03	.68	10.64	1.77	4.56	1.40	.61	2.24
Total	78,436								

*$\bar{R}$ = Average Requisition Size

Source: G570T
4/16/77

Analysis of 12,207
SM-ALC items; All
items with Demand > 0
in Jul-Sep 1976

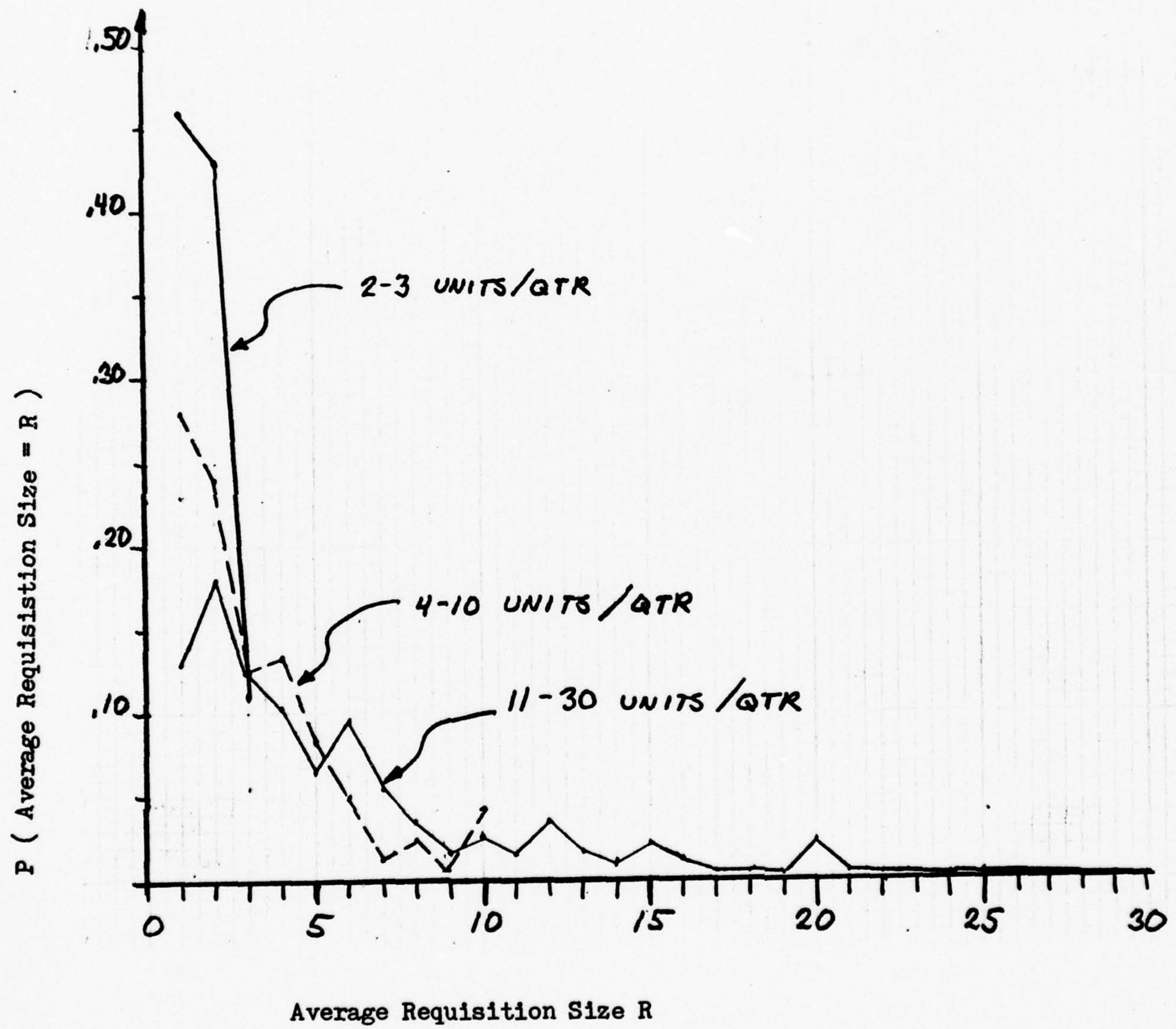


Figure II-1. Probability Distribution of Average Requisition Size.

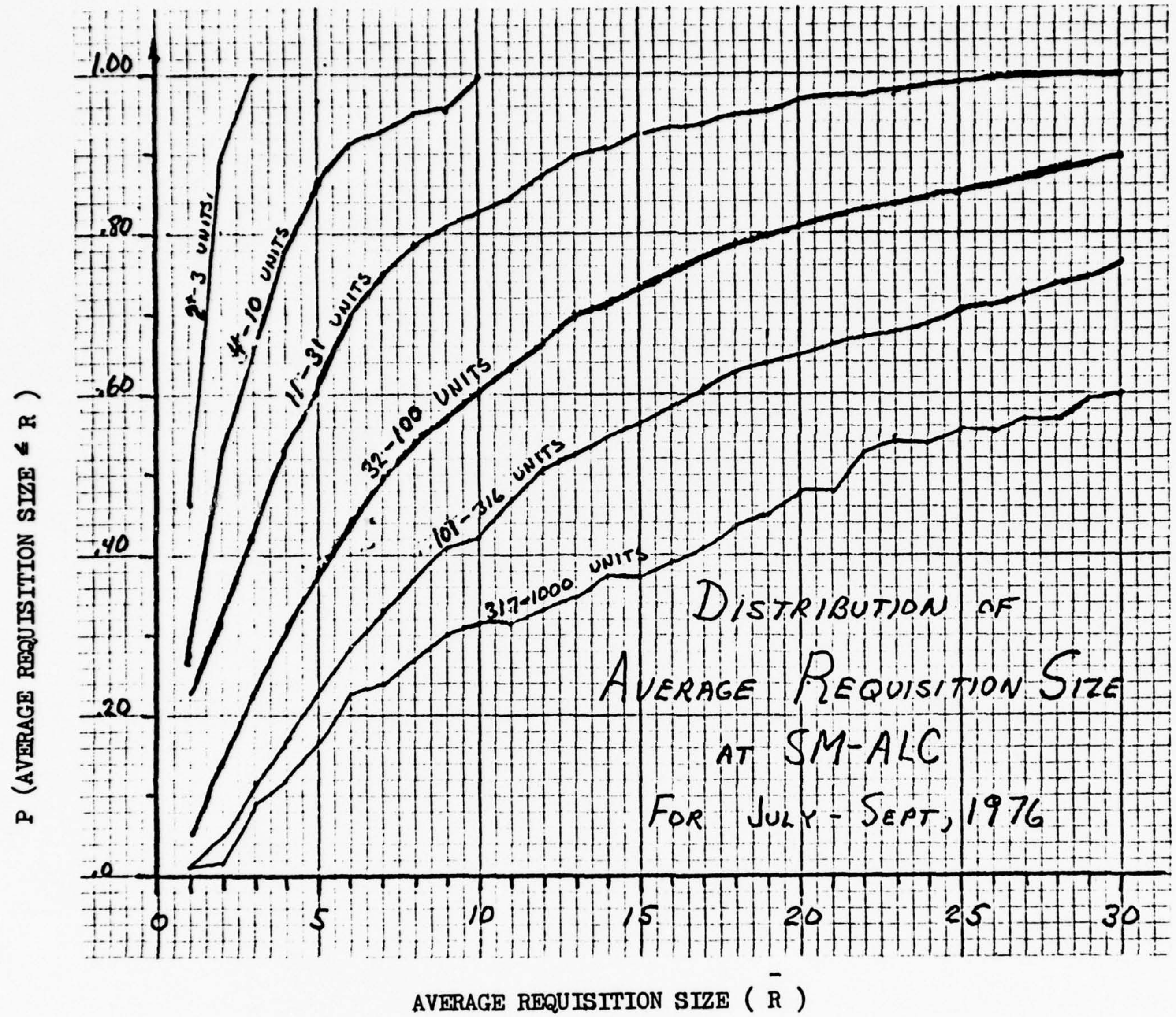
Distribution of Average Requisition Size

Of major importance in this study is the distribution of average requisition size. Unfortunately, only one period of data was available -- namely, July through Sept 1976 -- in which we were confident in the accuracy of both unit demand and unit requisition data. Consequently, it was impossible to develop statistical models to describe the probability distribution of requisition demand for individual items. It was possible, however, to develop the distribution of average requisition size as a function of specific item physical characteristics. A number of statistical studies were conducted in an attempt to relate the average requisition size for a specific item to its physical characteristics, such as unit cost, unit demand rate, item lead time, and dollar demand rate. The strongest relationship we found was between average requisition size and the unit demand rate of the item.

Figure II - 1 illustrates our results. As shown in the figure, if the unit demand for an item in the July through Sept period was two or three units, there was a 46% chance that the average requisition size for that item was one, while there was a 43% chance of an average requisition size of two units. Finally, we observed a 11% chance that the requisition size was 3. Two other curves, corresponding to 4-10 units/QTR and 11-30 units/QTR, are also shown in Figure II-1.

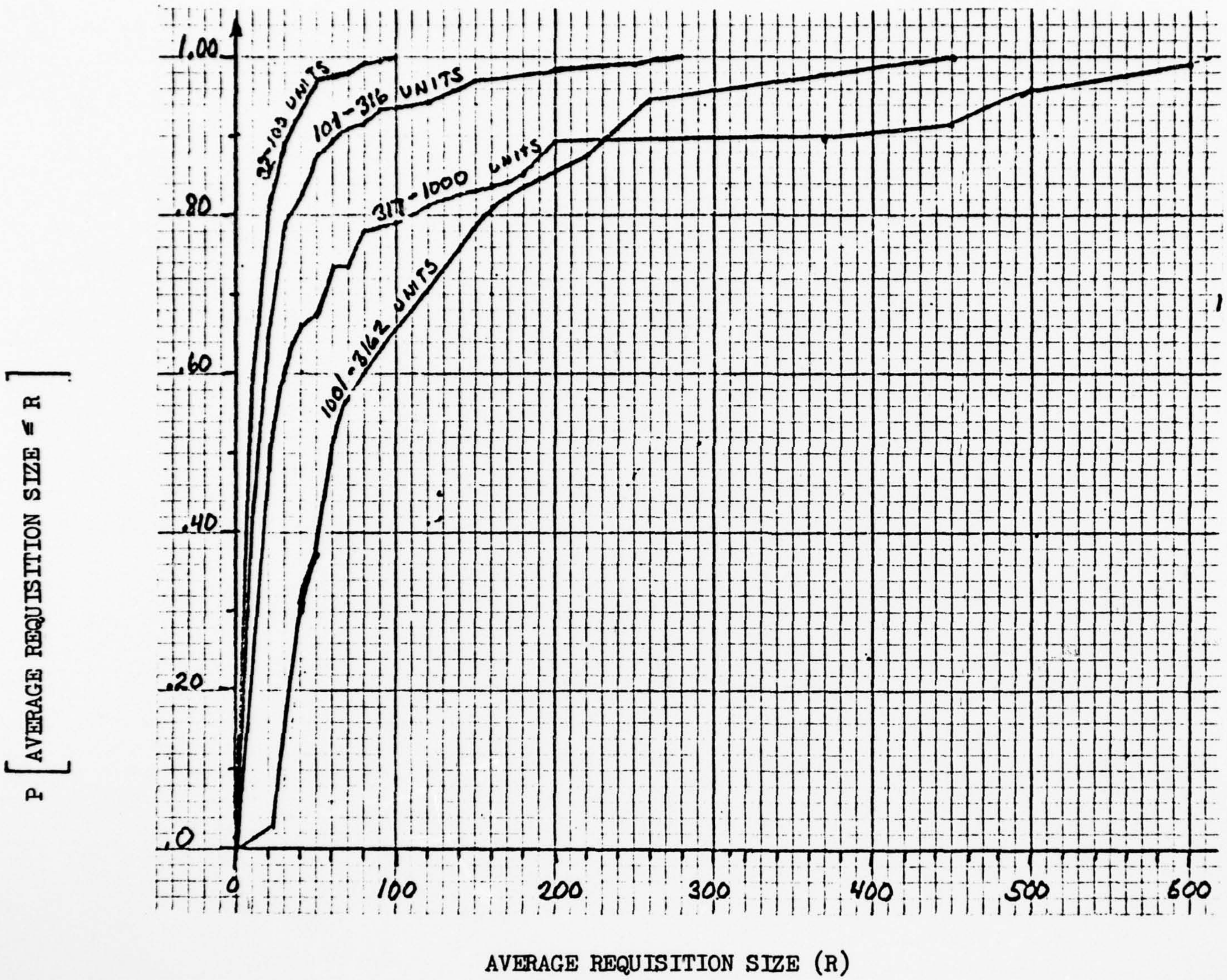
Figure II - 2 presents the distribution of average requisition size in the form of cumulative probabilities. For example, for an item with 2 or 3 units demanded in the July - Sept period, we observed a 46% chance of a requisition size of one, and an 89% chance (46% + 43%) that the average requisition size was less than or equal to two. Finally, if we restrict our attention to items with less than three units of demand, average requisition size must be less than or equal to 3; hence, there is a 100% probability that requisition size for these items is less than or equal to 3.

Figure II - 2 plots the cumulative probability functions of average requisition size for several classes of unit demand. As shown in the figure, as the number of units demanded within a period increases the probability of larger requisition sizes also increases. Recall, however that over 90% of all items have a quarterly demand of less than 31 units. Consequently, the average requisition size for most items will be small.



Demand in QTR	Number of Items
0	66,229
1	3,313
2-3	2,873
4-10	3,130
11-31	1,717'
32-100	819
101-316	245
317-1,000	67
1000+	23
Total	78,436

Figure II-2. Distribution of Average Requisition Size.



Demand in QTR	Number of Items
32-100	819
101-316	245
317-1,000	67
1.001-3,162	16
3,163+	7
Total	78,436

Figure II-2. Distribution of Aberage Requisition Size (continued).

II-16

Section III. Analytic Comparisons

The three formula sets defined in Appendix A share a common theoretical base. In this section, we first review this theoretical foundation, and we then explore the computational characteristics of these formulas. We are primarily interested in obtaining answers to questions such as:

> What are the computational differences among these formulas. Are there certain situations in which all of the formulas give essentially the same result?
>
> How sensitive is total system costs to departures from the optimum solution? How do the optimum values change as item characteristics change?
>
> How would aggregate inventories vary among these formulas? Would low moving items receive high safety stocks under one of the formulas, and low stocks under another? Or are there any differences?

Before we explore these issues, let us review the theoretical basis of the Presutti-Trepp formulas.

Presutti and Trepp studied a single-echelon, multi-item, continuous review inventory system in which the process generating demand does not change with time and each demand is for a single unit, i.e., the requisition size is one. When an item's reorder level r is reached, an order for Q units is placed. After a lead time, which is assumed constant, the entire order is received. For each item, it is desired to determine the constant reorder quantity Q and reorder point r which will minimize the

holding and procurement costs for the entire system subject to an overall backorder constraint. The following symbols apply throughout this section:

Q = order quantity (units)

r = reorder level (units)

D = annual demands (units)

k = safety factor

σ = standard deviation of demand during a lead time

a = holding cost factor

A = ordering cost (dollars per order)

c = item cost (dollars per unit)

x = number of units demanded in a lead time (a random variable)

u = mean lead time demands (units)

W = item essentiality (relative military worth)

In addition, it is assumed that the reorder point is set equal to the expected lead time demand plus a safety stock; specifically,

$$r = u + k\sigma$$

where $k\sigma$ is the safety stock.

Presutti and Trepp assume that the probability density function (p.d.f.) describing the distribution of the number of units demanded during a lead time is given by

$$\text{III-1} \quad f(x) = \frac{1}{\sqrt{2}\sigma} \exp\left(-\sqrt{2}\left|\frac{x-u}{\sigma}\right|\right)$$

for all X. They point out that this function is quite similar to the normal p.d.f., but is easier to deal with mathematically. They then show that P_{out}, the probability of a stockout at a randomly-selected point in time, is given by

$$\text{III-2} \quad P_{out} = \frac{1}{2\sqrt{2}} \frac{\sigma}{Q} (1 - \exp(-\sqrt{2}\, Q/\sigma)) \exp(-\sqrt{2}\, k)$$

where Q is the order quantity and k denotes the number of standard deviations that the reorder point r exceeds the expected lead time demand, i.e.,

$$\text{III-3} \quad k = \frac{r - u}{\sigma}$$

They then observe that once P_{out} is known, it is easy to compute EBO, the expected number of units backordered at a random point in time, since

$$\text{III-4} \quad EBO = \frac{\sigma}{\sqrt{2}} P_{out}$$

Presutti and Trepp then consider the problem of computing optimal order quantities and reorder points in multi-item inventory systems under several different cost structures and service policies. In Model IV, they consider the problem of computing values of Q_i and k_i that minimize the total expected costs of ordering and holding inventory subject to a constraint on the essentiality-weighted backorders associated with the system. Mathematically, the problem is to minimize TC,

$$\text{III-5} \quad TC = \sum_{i=1}^{I} \frac{A_i D_i}{Q_i} + \sum_{i=1}^{I} a_i c_i (u_i + k_i \sigma_i + \frac{Q_i}{2})$$

subject to

$$\text{III-6} \qquad \sum_{i-1}^{I} W_i \, EBO_i(Q_i, k_i) \leq B$$

We will refer to this as problem P1.

The first term of III-5 is the expected annual cost of ordering inventory, while the last term of III-5 denotes the expected annual cost of carrying this stock (assuming carrying cost is proportional to the total stock on-hand and on-order in the system). In constraint III-6, W_i denotes a military essentiality factor, or weight, that measures the severity of a unit backorder of item i relative to a unit backorder of some other item. The constraint III-6 thus represents a limit on the essentiality-weighted back-order-days expected in the inventory system as a whole.

If problem P1 is solved using Lagrange Multipliers, we consider the problem of minimizing $\overline{TC}$,

$$\text{III-7} \qquad \overline{TC} = \sum_{i=1}^{I} \frac{A_i D_i}{Q_i} + \sum_{i=1}^{I} a_i c_i \left(u_i + k_i \sigma_i + \frac{Q_i}{2}\right) + \sum_{i=1}^{I} \lambda W_i EBO_i(Q_i, k_i)$$

where λ is a new decision variable with the dimensions of dollars per backorder-year. Hence, if λ is known, TC is conceptually equivalent to the unconstrained problem of minimizing the sum of ordering, holding <u>and</u> <u>shortage</u> costs. We refer to this Lagrangian problem as problem P2.

Taking derivatives of III-7 with respect to Q_i and k_i, respectively, it may be shown that the optimal values may be approximated by

$$\text{III-8} \quad \begin{cases} k_i = \dfrac{1}{\sqrt{2}} \ln \left\{ \dfrac{\lambda W_i \sigma_i \left(1 - \exp(-\sqrt{2}\; Q_i/\sigma_i)\right)}{2\sqrt{2}\; Q_i a_i c_i} \right\} \\ Q_i = \sqrt{\dfrac{2A_i D_i}{a_i c_i}} \end{cases}$$

provided that $\exp(-\sqrt{2} Q_i/\sigma_i)$ is very small. Note that the equation for Q is the well-known Wilson lot size.

Suppose that W_i, the military essentiality associated with item i, is inversely proportional to the factor Z, i.e., suppose W = 1/Z. In this case, the optimal safety factor k may be written as

$$\text{III-9} \quad k = \frac{1}{\sqrt{2}} \ln \left[\frac{\lambda_i}{2a_i c_i} \cdot \frac{1}{Z_i} \cdot \frac{\sigma_i \left(1 - \exp(-\sqrt{2}\; Q_i/\sigma_i)\right)}{\sqrt{2}\; Q_i} \right]$$

This is the same as equation B(6) in Appendix A. Note that Z = 1 implies that each unit backorder has the same military essentiality, while $Z = \bar{R}$ = (average requisition size) implies that a unit backorder for a item with a high average requisition size is not as critical as a unit backorder for an item with small average requisitions.

Also, observe that the optimal values for a given item i are a function of the parameter λ and other parameters associated with item i. Hence, given λ, optimal values of Q_i and k_i may be computed without any knowledge of the parameters of other items. Hence, when we are discussing a specific item i, we may drop the i subscript without any loss of information. We shall do this in the following discussions.

Solving Problem P1

Given III-8 we may then solve problem P1 approximately by first solving problem P2 for a series of values of λ. The particular set of (Q_i, k_i) that minimize TC while still satisfying III-6 should then provide a good approximate solution to the problem P1.

Let us now investigate the analytical properties of these formulas.

Order Quantity Sensitivity

Formula III-8 yields the optimum order quantity in units. This quantity may be expressed in terms of months of supply by dividing Q by the monthly demand rate (D/12). This gives

$$\text{III-10} \quad \begin{cases} Q_{\text{months}} = Q \div (D/12) \\ \quad = \sqrt{\dfrac{24\,A}{a} \cdot \dfrac{1}{cD}} \end{cases}$$

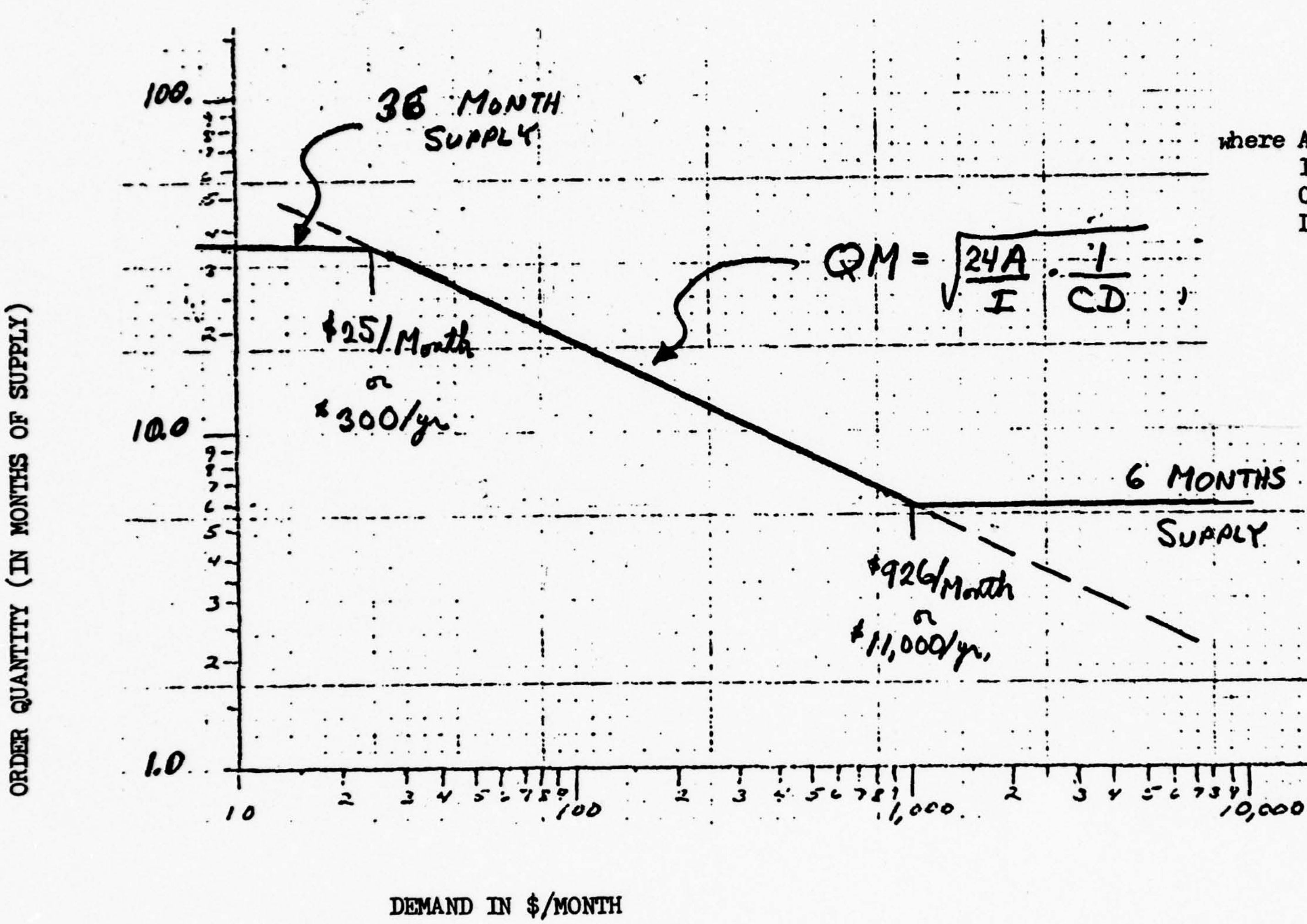
Hence, the optimum order quantity in months of supply will increase if the set-up cost A is increased, and it will become smaller if there are increases in the holding cost (a), the item unit cost (c), or the item demand rate (D).

Observe that cD is the annual dollar demand rate of the item. Hence, the optimal order quantity in months of supply is inversely proportional to the square root of the annual demand rate. Figure III-1 plots this relationship using

A = \$278/order

a = \$.20 carrying cost for each \$1 of inventory held for one year.

ORDER QUANTITY
VS
DOLLAR DEMAND RATE



where A=$278/order
I=$.20/$ inventory per year
C= unit cost
D= monthly demand rate

Figure III-1. Order Quantity in Months of Supply verses Dollar Demand Rate.

These are the current cost parameters used in the EOQ Buy Computation System (D062) for items with annual demands of \$10,000 or less.

As shown in the figure, the order quantity Q_{months} decreases as annual dollar demand increases. If the annual demand value cD exceeds \$11,000 per year, or \$926/month, the formula III-10 yields order quantities that are less than 6 months of supply. Conversely, for items with annual demands of less than \$300/year or \$25 per month, the formula yields order quantities that exceed a 36 month supply.

At present, the D062 system limits all order quantities to be no more than a 36 month supply, and no less than a 6 month supply. Hence, the solid line in Figure III-1 plots the order quantities computed in the D062 system.

Safety Factor Sensitivity

Observe that the Lagrangian III-7 is simply the sum of the expected costs associated with each item in the system (including implied shortage costs.) If the order quantity Q is fixed, the variable costs, V, associated with a given item may be written as

III-11 $$V = ack\sigma + \lambda W \cdot EBO(Q,k)$$

We may normalize this expression by dividing each side by $ac\sigma$, the cost of carrying for one year an amount of inventory equal to one standard deviation of lead time demand. The result is

$$\text{III-12} \quad \frac{V}{ac\sigma} = k + \frac{\lambda W}{ac\sigma} \cdot EBO(Q,k)$$

Recall, however, that

$$\text{III-13} \quad EBO\ (Q,\ k) = \frac{\sigma}{\sqrt{2}} P_{out}$$

Hence, if we substitute III-13 and III-2 in equation III-12, we obtain

$$\text{III-14} \quad \frac{V}{ac\sigma} = k + \frac{\lambda W}{ac} \cdot \frac{\sigma}{4Q} \left[1 - \exp(-\sqrt{2}\ Q/\sigma)\right] \exp(-\sqrt{2}\ k)$$

Now let

$$\text{III-15} \quad \begin{cases} X = \sqrt{2}\ Q/\sigma \\ Y = \dfrac{1 - \exp(-X)}{X} \end{cases}$$

Substituting in III-14, we obtain

$$\text{III-16} \quad \frac{V}{ac\sigma} = k + \frac{\lambda W}{ac} \cdot \frac{Y}{2\sqrt{2}} \cdot \exp(-\sqrt{2}\ k)$$

The factor Y represents an adjustment to correct expected backorders for the "hidden safety stock" that is implicit in large order quantities. Table III-1 illustrates the relationship among Y and the ratio Q/σ. Observe that as the order quantity Q becomes large relative to σ, the backorder adjustment Y decreases significantly.

Table III-1

Q/σ	$X = \sqrt{2}Q/\sigma$	$Y = (1.-\exp(-X))/X$
.01	.0141	.9930
.05	.0707	.9655
.10	.1414	.9325
.50	.7070	.7169
1.00	1.4140	.5352
1.50	2.1210	.4149
2.00	2.8280	.3327
2.50	3.5350	.2746
3.00	4.2420	.2323
3.50	4.9490	.2006
4.00	5.6560	.1762
4.50	6.3630	.1569
5.00	7.0700	.1413

Robert J. Stevens has observed (See AFLC/ACDRL Working Paper No. 36, December 1971) that for the 1969-1971 time period, the standard deviation of annual unit demand for EOQ items is related to the annual unit demand rate D by the regression equation

$$\bar{\sigma} = .904\ D^{.9}$$

Hence, for an "average" item with a 12 month lead time and an order quantity of 6 months supply, Q = .5D, which gives

$$Q/\sigma = .5D \div (.9D^{.9}) = .55D^{.1}$$

On the other hand, if the order quantity for such an item is a three year supply, Q = 3D, and

$$Q/\sigma = 3D \div .9D^{.9} = 3.3\ D^{.1}$$

In Table II-1, we observed that 99% of all SM-ALC items have less than 100 demands per quarter, or 400 unit demands per year. For an item with D = 400, we obtain $3.3D^{.1} = 6.0$. Hence, for an "average" item with a 12 month lead time and an order quantity of 6 months supply, $Q = .5D$, which gives

$$Q/\sigma = .5D \div (.9D^{.9}) = .55D^{.1}$$

Hence, for such an "average" item, we would expect Q/σ to be in the range of .5 to 6, depending on the value of D. For items with shorter lead times, we would expect Q/σ to be slightly higher, since the value of σ would be less.

Now let us define

$$\text{III-17} \qquad F = \frac{\lambda W}{ac}$$

The numerator of F, λW, may be interpreted as the relative military cost of having one unit in a backorder status for one year. On the other hand, the denominator, ac, is the cost of carrying one unit in inventory for one year. Hence, F denotes the ratio of annual shortage costs to annual carrying costs for a single unit.

Finally, substituting III-17 into III-16, we obtain

$$\text{III-18} \qquad \frac{V}{ac\sigma} = k + F \cdot \frac{Y}{2\sqrt{2}} \cdot \exp(-\sqrt{2}\ k)$$

Equation III-18 describes the expected annual costs of carrying safety stock and of shortages, expressed as a

multiple of $ac\sigma$, the cost of carrying for one year an amount of stock equal to one standard deviation of lead time demand.

Figure III-2 illustrates the behavior of total annual cost as a function of the safety factor k for several values of the parameter F, and for X = .1. Observe that as the safety factor k is increased, the carrying cost increases linearly, but the shortage cost decreases exponentially. Eventually, a minimum point is reached. In Figure III-2, a star "★" denotes the minimum cost for an item with a given F-value, and the straight line connecting the stars is the plot of the minimum annual cost and associated k-values for all values of F with X = .1. As may be seen in the figure, as F increases, the optimum safety factor also increases.

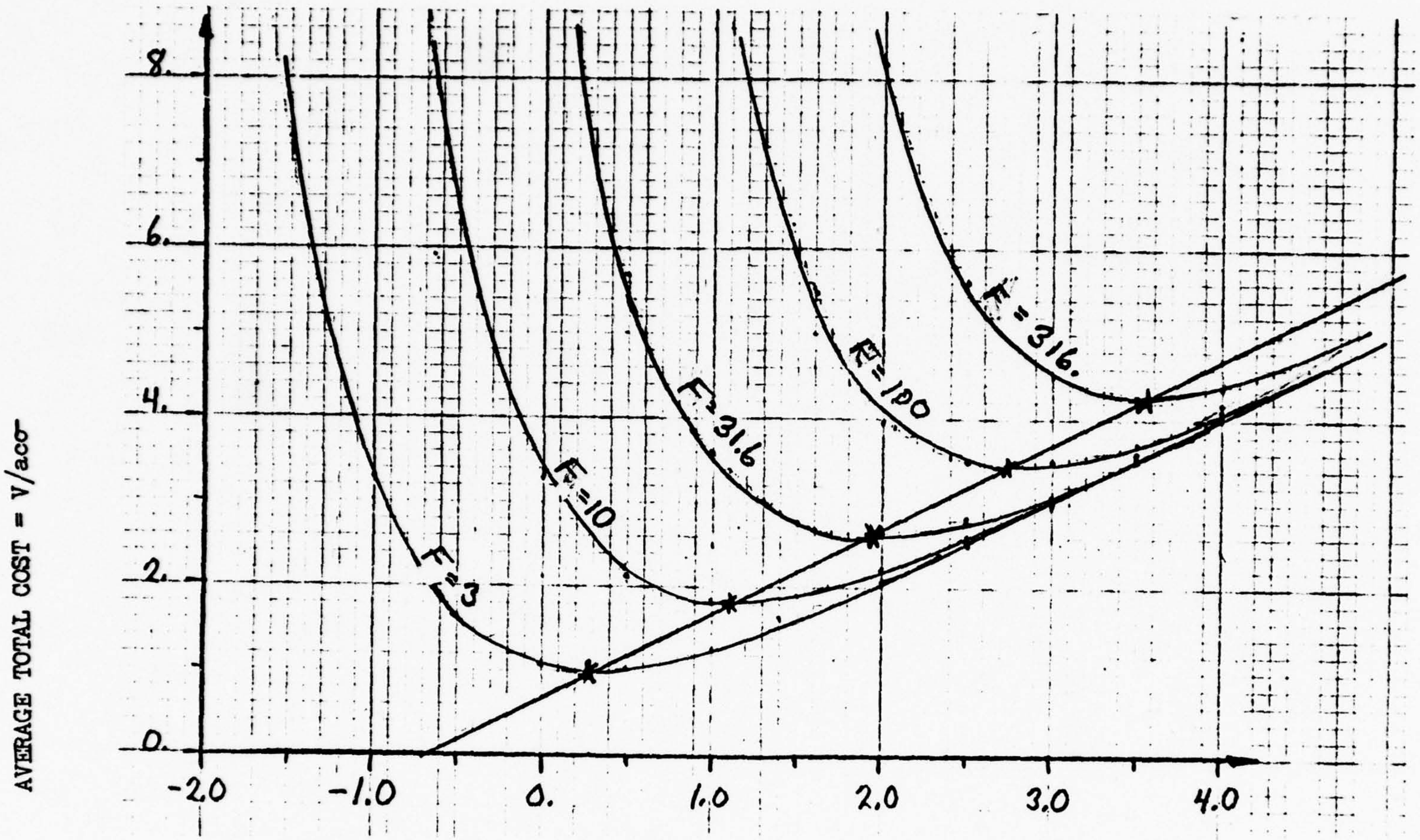
Notice that the total cost curve is fairly flat near the optimum k value. Hence, total costs will be very low as long as safety stocks are in the general neighborhood of the optimal value.

The optimum safety factor may be restated by substituting III-15 and III-17 into III-8, yielding

$$k = \frac{1}{\sqrt{2}} \ln \left[\frac{F}{2} \frac{(1 - \exp(-X))}{X} \right]$$

This curve is plotted in Figure III-3 for a series of x values. As we observed earlier, the ratio of Q/σ should

$$\frac{\tilde{V}}{ac\sigma} = k + F \cdot \frac{Y}{2\sqrt{2}} \exp(-\sqrt{2}\,k)$$



SAFETY FACTOR (k)

Figure III - 2 Average annual Cost versus Safety Stock

X = .1

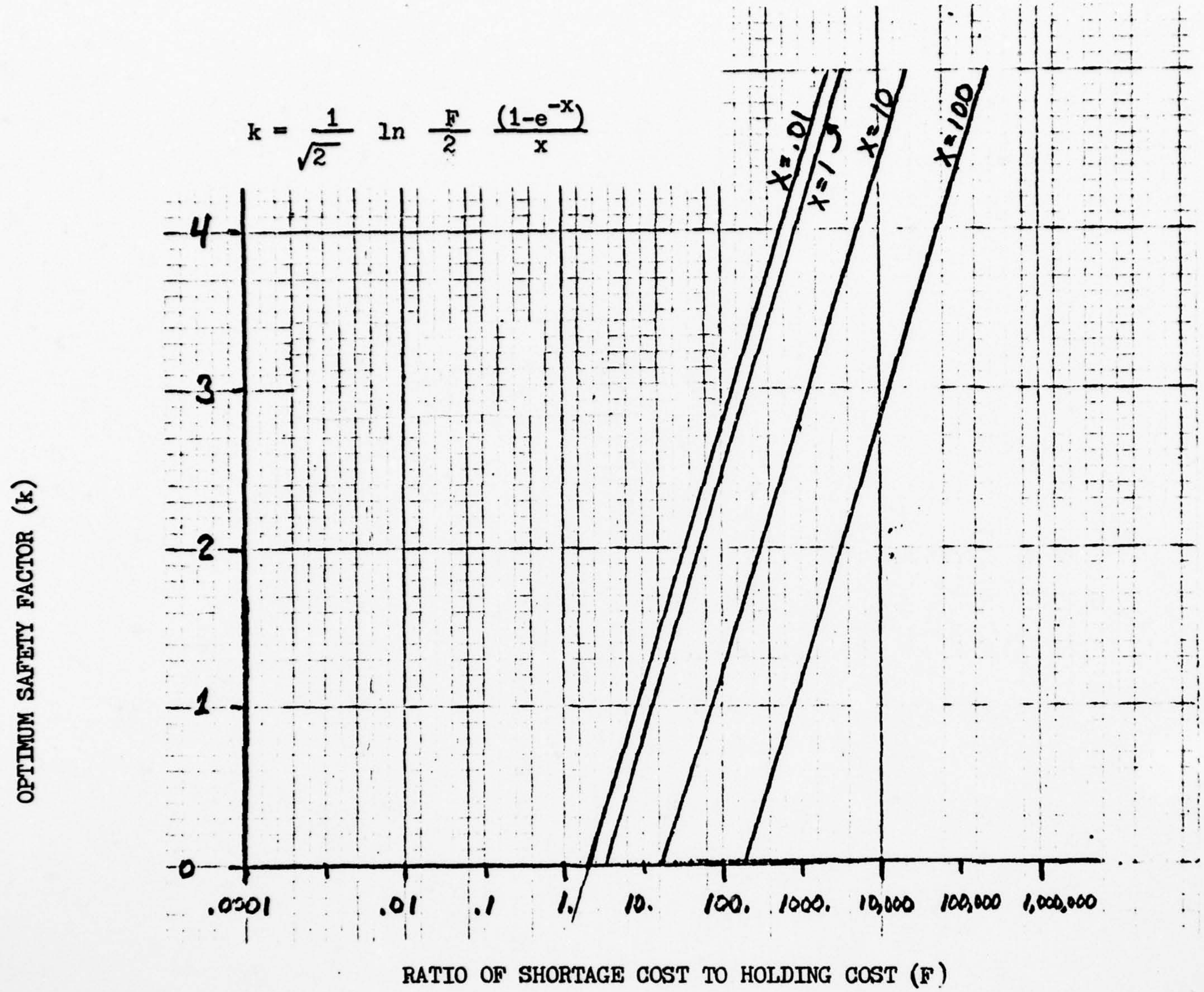


Figure III -3 Optimum Safety Factor versus F

lie in the range of .5 to 6 for many items. Since $X = \sqrt{2}Q/\sigma$, this corresponds to a range of X values of .7 to 8.

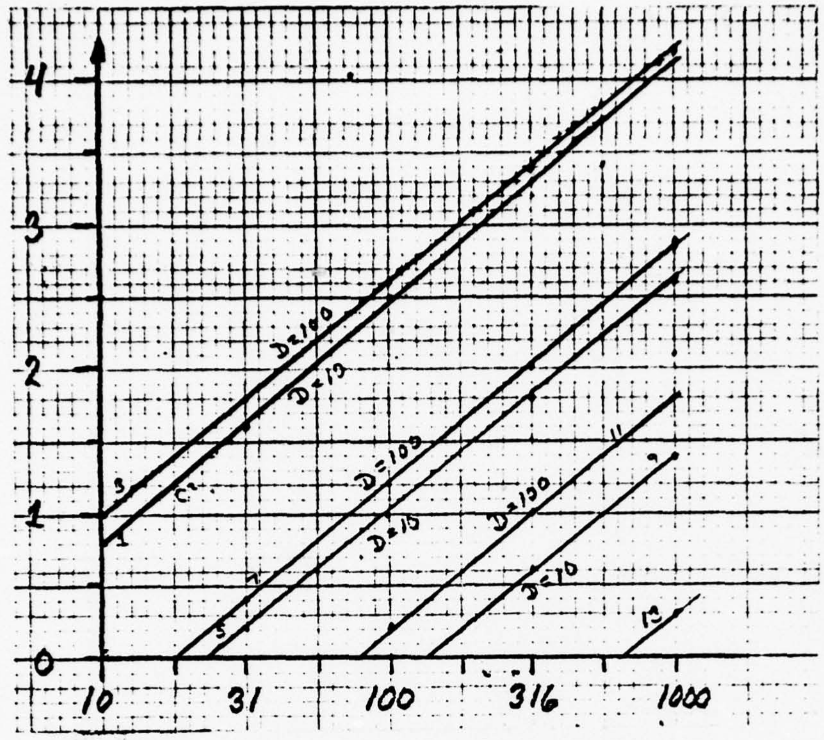
Sensitivity to Unit Cost and Unit Demand Rates

Figures III-4 and III-5 illustrate the sensitivity of the PT-formulas to varying unit costs and unit demand rates. These figures present results for six hypothetical items with the following characteristics:

Item	Unit Cost ($)	Annual Demand Rate (Units)	Std Deviation of Demand in Leadtime
1	$ 1	10	7
2	10	10	7
3	100	10	7
4	1	100	57
5	10	100	57
6	100	100	57

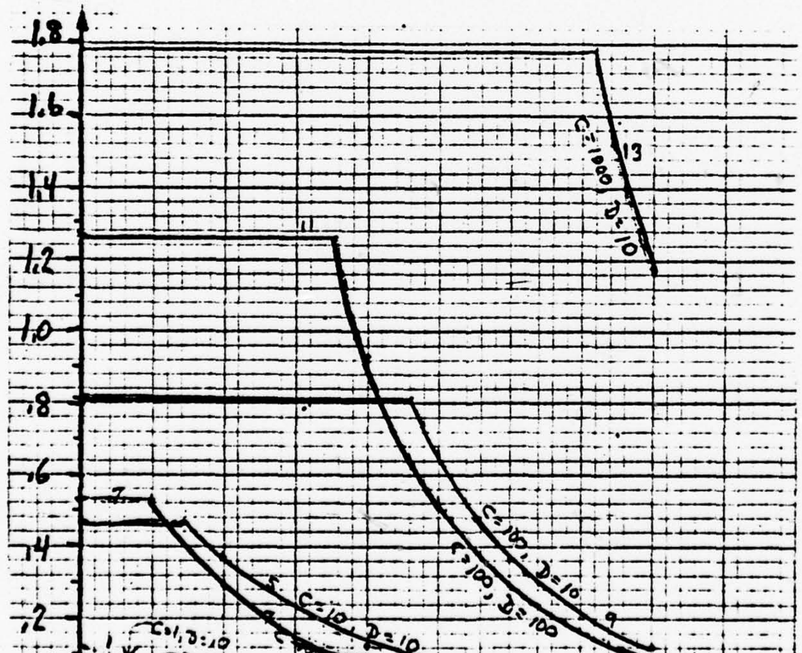
Figure III-4 plots the relationship among the shortage cost λW and the optimum safety factor k, while Figure III-5 presents the associated cost-effectiveness curves for these items.

SAFETY STOCK FACTOR (k)



SHORTAGE COST ( )W)

UNIT BACKORDERS IN MONTHS OF SUPPLY



SHORTAGE COST ( )W)

Figure III - 4 Backorders verses Short Cost.

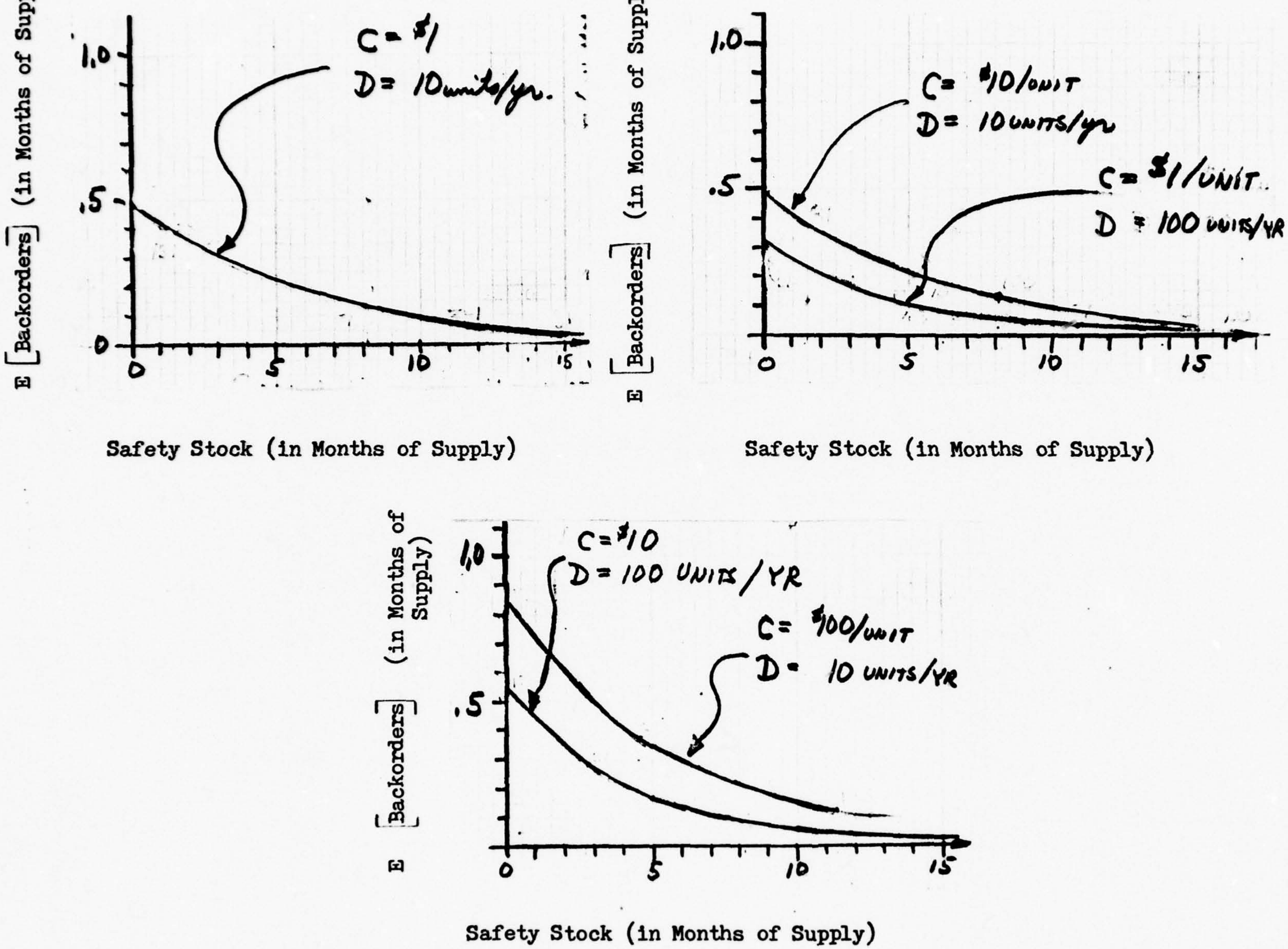


Figure III-5. Backorders verses Safety Stock Curve.

Section IV. The Simulation Model

The anlytical investigations presented in Section III provide important insights into computational differences among the alternate Presutti-Trepp formulas. However, analytic studies are of limited usefulness in studying the relative cost-effectiveness of these formulas for use in the EOQ Buy Computation System (D062). This is because the validity of analytic projections require that the demand process remain stationary through time. In real systems, conditions always change as time passes. The important test is how well a particular set of inventory control rules will behave in the dynamic environment in which they are to be used. For such evaluations, simulation techniques are required.

To evaluate the relative cost-effectiveness of the alternate PT-formulas, we needed a detailed simulation model of the D062 system. Before we discuss how this model was developed, let us first review the major features of EOQ Buy Computation System (D062).

The EOQ Buy Compuation System

The EOQ Buy Computation System (D062) is the primary data system for the control of secondary items managed by the Air Force Logistics Command. In general, the term "secondary item" refers to a discrete component that is part of a higher assembly and is not economically reparable at the depot level of supply. The terms "repair parts," "bits and pieces," "consumable items," "stock fund items," and "expense items" are often used to describe these parts. AFLC manages approximately 500,000 of these items.

The primary functions of the EOQ Buy Computation System are to:

a. Accumulate demand data.

b. Compute depot stock levels.

c. Determine buy, termination, and long supply quantities.

d. Provide a baseline for funds projections.

e. Provide reports and management data.

As illustrated in Figure IV-1, inputs to D062 come from several sources. Headquarters AFLC specifies the implied shortage factor (λ) required by the Presutti-Trepp safety stock formula. Stock list data, asset and usage counts, and file maintenance actions are other inputs to the D062 system.

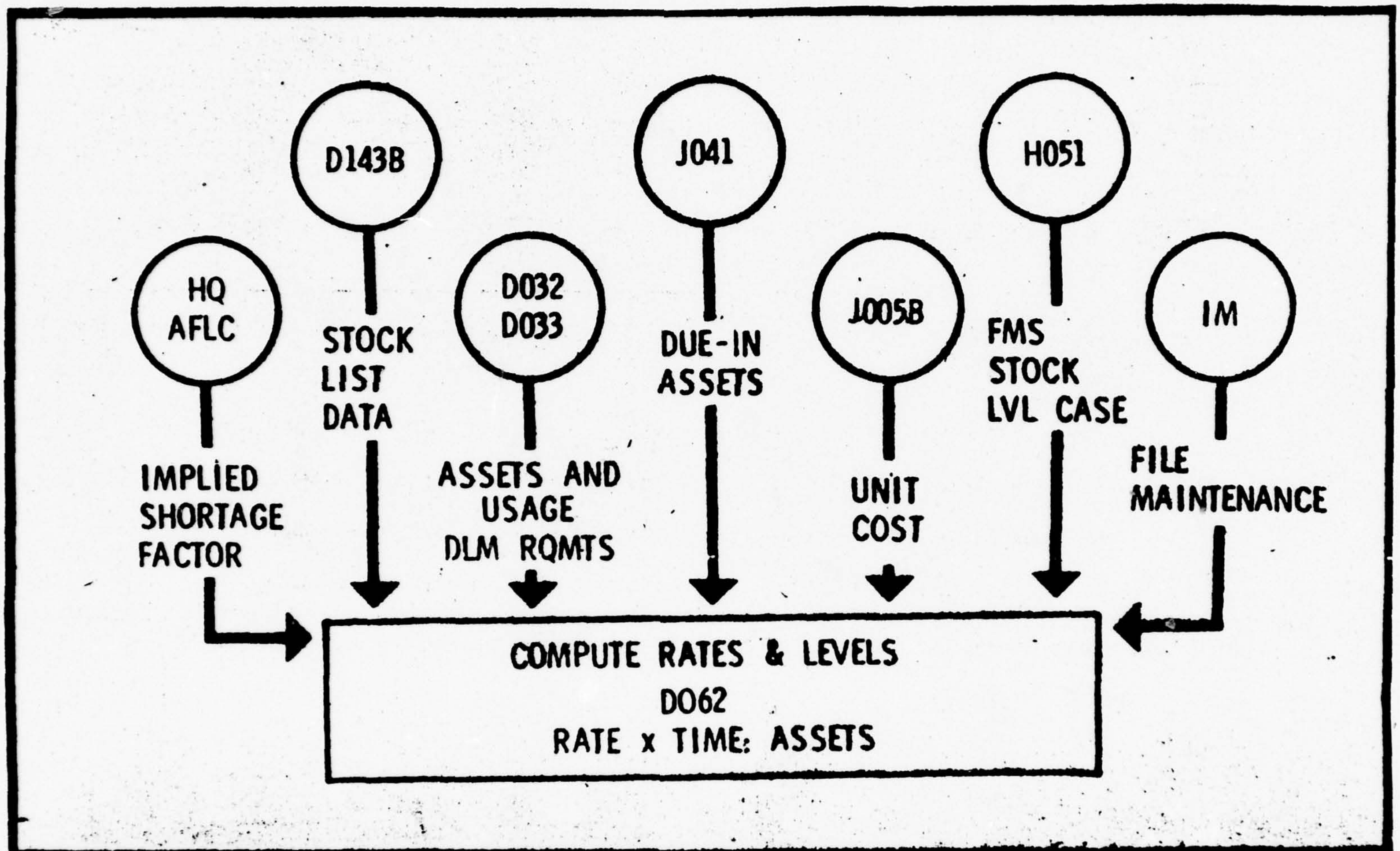


Figure IV-1. Inputs to the Requirements Process

Note: The HOSI and DLM RQHTS inputs are not complemented at present.

This information is used to compute several critical numbers, called "levels," that are used to initiate management notices on a by exception basis. For example, currently available assets are compared to the levels to determine if buy, termination, or disposal actions are needed.

Figure IV-2 illustrates the major outputs of the D062 System. These include:

a. Requirements Notices -- The item manager receives advance buy, buy, or termination notices when assets breach the respective levels. Item interrogations can also be requested by the item manager.

b. Management Reports -- These data products are produced for the Air Logistics Centers and Headquarters AFLC summarizing the impact of the computation by categorizing items according to actions required.

c. CSIS Data -- Data required to perform the Central Secondary Item Stratifaction is passed to the D075 system every quarter.

d. Data to Other Interfacing Systems -- D062 also provides information to the D067 and D032 systems. D067 is furnished data required to process excesses, and D032 is fed control levels required for distribution of assets.

Control Levels

In the D062 system, the terms "asset position" and "inventory position" refer to the total assets on hand and on order in the system, less any backorders. Hence, an item's asset position is the total stock available to meet future demands if there are no more buys.

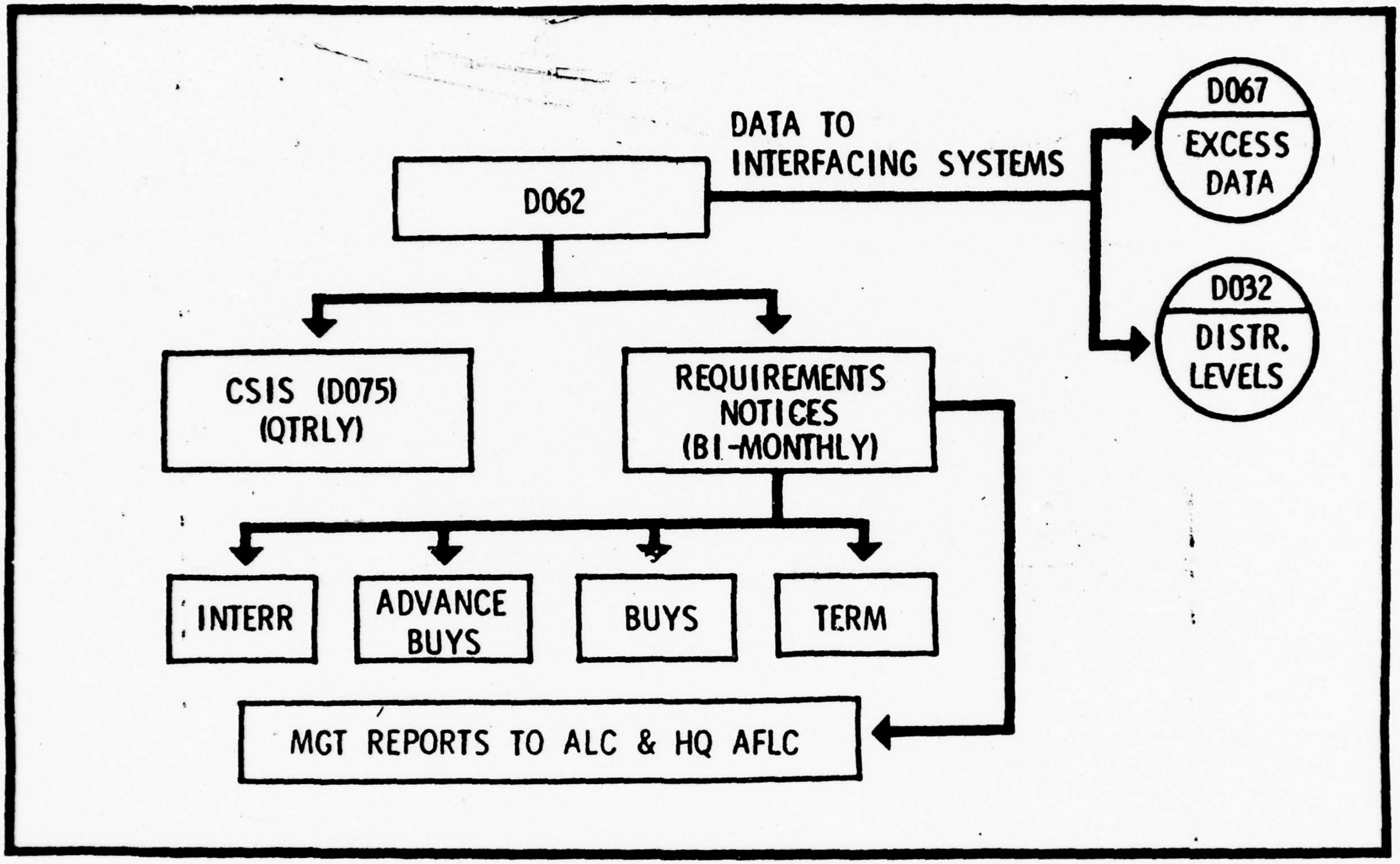


Figure IV-2. Outputs from the EOQ Buy Computation System

As noted above, D062 computes various "levels" which are compared against an item's asset position to determine if any action is required. These levels are summarized in Figures IV-3 and IV-4. Procurement lead time is the average time that elapses from the first printout of a buy notice to the date of the first significant delivery. The safety level is variable depending on item characteristics and HQ AFLC inputs. Its purpose is to insure continuous operation in the event of unpredicted fluctuation in demand and/or extended lead times.

The reorder level (ROL) equals the number of demands expected in the procurement lead time plus the safety level. It is the point to which available assets are compared to determine if a buy action is required. When an item's inventory position equals or falls below the reorder level, the buy quantity consists of any deficiency to the reorder level plus an economic order quantity.

The data level represents four months of demands beyond the reorder level. The function of the data level is to provide early identification of items in a potential buy position.

The termination level is one years worth of demands beyond the ROL. Note: If EOQ $<$ one year's supply, the termination level = ROL + EOQ. Termination level notices

PROCUREMENT LEAD TIME (PLT)

- TIME FROM PREPARATION OF PR / MIPR TO DATE OF FIRST SIGNIFICANT DELIVERY

SAFETY LEVEL

- MATERIEL AUTHORIZED TO BE ON HAND TO INSURE CONTINUOUS OPERATION IN THE EVENT OF UNPREDICTED FLUCTUATION IN DEMANDS AND / OR EXTENDED LEAD TIME

REORDER LEVEL

- COMPUTED POINT TO WHICH ASSETS ARE COMPARED TO DETERMINE IF BUY ACTION IS REQUIRED

DATA LEVEL

- PROVIDES EARLY IDENTIFICATION OF ITEMS IN A POTENTIAL BUY POSITION

Figure IV-3. Levels

TERMINATION LEVEL (TL)

- USED TO DETERMINE THOSE ITEMS WHICH ARE CANDIDATES FOR CONTRACT TERMINATION

APPROVED FORCE ACQUISITION OBJECTIVE (AFAO)

- DETERMINES BEGINNING OF ECONOMIC RETENTION
- USED AS STOCK FUND CREDIT INDICATOR

RETENTION LEVEL

- INDICATES MAXIMUM LEVEL OF STOCKS WHICH MAY BE RETAINED IN SUPPLY SYSTEM. (CONSISTS OF AFAO 1 - 5 YEAR'S REQMTS.)

PROJECTED NET BUY

- ROL DEFICIENCY + EOQ

Figure IV-4. More Levels

are output for items with on order assets when the total asset position is greater than the termination level. This notice warns the item manager that it may be desirable to cancel delivery of at least part of the on order quantities. The approved force acquisition objective (AFAO) consists of two years worth of forecast demands plus the lead time and safety level requirements. (Note: If EOQ $>$ 2 years of supply, AFAO = EOQ + ROL.) Items with assets greater than The AFAO are considered to be in long supply.

Finally, the retention level indicates the maximum amount of stock which may be retained in the supply system. Generally, quantities beyond this value are considered excess. Retention levels can vary from one to five years of projected demands beyond the AFAO. Retention levels for items supporting newer weapon systems generally use five years, while systems ready to be phased out may have retention levels equal to the AFAO plus one year of projected demands.

Formulas

A basic problem in any inventory system is determining how much stock should be on hand. If too much stock is procured, excessive carrying costs are incurred. On the other hand, if too little stock is procured, an item must be procured more often, and excessive procurement costs are

incurred. In the D062 system, the Wilson lot size formula is used to obtain a good balance between these conflicting costs. This formula takes the form:

$$Q = \sqrt{\frac{2AD}{H}}$$

where Q equals the order quantity in units, D is the annual demand rate in units, A is the cost per order placed, and H is the cost of holding one unit in inventory for one year. At present, a holding cost H of 20% of the item unit cost per year is used in the lot size formula.

In Air Force supply systems, different procurement methods are employed for small purchases than are used for high dollar buys. Simplified procurement techniques are used for small dollar purchases. These methods may be used for purchases of less than $10,000. On the other hand, advertized, negotiated contracts are used for high dollar buys. At present, the following order costs are used in the D062 system:

<u>COST TO ORDER</u>

$269.87	for purchases of less than $10,000
$460.27	otherwise

In applying the EOQ formula above, impractical values for order quantities are produced for items with very high

or with very low annual demand rates. Consequently, EOQ's are bounded to be no more than a 36 month supply, and no less than a 6 month supply.

At present, safety levels are computed in D062 using the Presutti-Trepp formula with $Z = \sqrt{R}$. This quantity is then bounded to be no more than (a) the number of demands expected in the procurement lead time or (b) three times the standard deviation of lead time demand, whichever is smaller.

Figure IV-5 illustrates the levels computations for a particular, fast moving item. This item has a unit price of $10, a procurement lead time of nine months, and an average of 100 demands per month. The expected demand in the nine month procurement lead time is thus 9 X 100 = 900 units. The safety level for this particular item is 113. This was determined based on the PT-safety level formula. The reorder level is the sum of the safety level and the expected demand in the procurement lead time, which gives us 113 + 900 = 1013. The data level is four months worth of demands beyond the reorder level. This item has annual demands D of 1200 units per year, and the cost of holding one unit in inventory for one year is 20% X unit price = $2 per year. Hence, the EOQ is $\sqrt{2AD/H} = \sqrt{2 \times (\$269) \times (1200)/\$2}$, or 568 units. A cost per order of $269 is used in this calculation since the dollar value of the purchase

9413	RETENTION LEVEL	AFAO + 5 YRS
3413	AFAO (TERMINATION LEVEL)	ROL + 2 YR
568	EOQ	$\sqrt{\frac{2\,AD}{H}}$ (5.6 MOS)
1413	DATA LEVEL	ROL + 4 MOS
1013	REORDER LEVEL	SL + PLT
113	SAFETY LEVEL	VARIABLE
900	PROCUREMENT LEAD TIME	9 X MONTHLY DEMAND

<u>ITEM DATA</u>

UNIT PRICE $10

PLT 9 MOS

AV MOS DEMANDS 100

Figure IV-5. Levels Computation

is \$10 X 568 = \$5,680. Since this is less than \$10,000, small purchase methods may be used. The AFAO is two years of stock beyond the ROL and the retention level is five years worth of demands beyond the AFAO.

INSSIM - The Inventory System Simulator

As noted above, a detailed simulation model of the D062 system was needed to evaluate the relative effectiveness of the alternate PT-formulas. In our study, we utilized the Inventory System Simulator (INSSIM) as a starting point. This simulator was developed by the Directorate of Management Sciences (AFLC/XRS) to evaluate inventory policies in single location supply systems. For our study, it was necessary to enhance the original model to provide a detailed description of the current D062 system and to provide for improved input and output capabilities. In the following discussions, we will use the term "INSSIM" to refer to the enhanced version of the original simulator.

Major Features of the Inventory System Simulator (INSSIM)

The Inventory System Simulator (as enhanced) possesses the following major characteristics:

a. A detailed description of the EOQ Buy Computation System (D062).

b. A demand process based upon actual demand histories for Air Force items.

c. Comprehensive measurement of simulation results.

d. Entensive input options -- which permit the evaluation of several proposed forecasting and inventory control policies by simple changes to input data.

e. A modular structure -- to simplify future enhancements to the model.

f. Debugging aids -- to assist in the programming of proposed rules that are not already coded in the model.

Basically, INSSIM consists of a collection of Fortran subroutines and a MAIN program that controls input requirements, schedules events within the simulation, and initiates output products. The major INSSIM routines are illustrated in Figure IV-6, grouped by their major function. As shown in the figure, the major features of the D062 system are simulated using subroutines FOR576, FORUPD, LEVEL, and STATUS. The major functions of these routines are as follows:

Subroutine	Function	Description
FOR576	Forecasting	This routine provides estimates of ° gross demand rates ° serviceable return rates ° net demand rates ° average requisition size ° demand variability
FORUPD	Record demand history	This routine maintains an eight quarter moving history of simulated demand. This history is used in the forecasting calculations in FOR576.
LEVEL	Computes inventory control levels	This routine computes the inventory control levels discussed above (safety levels, reorder levels, etc.)

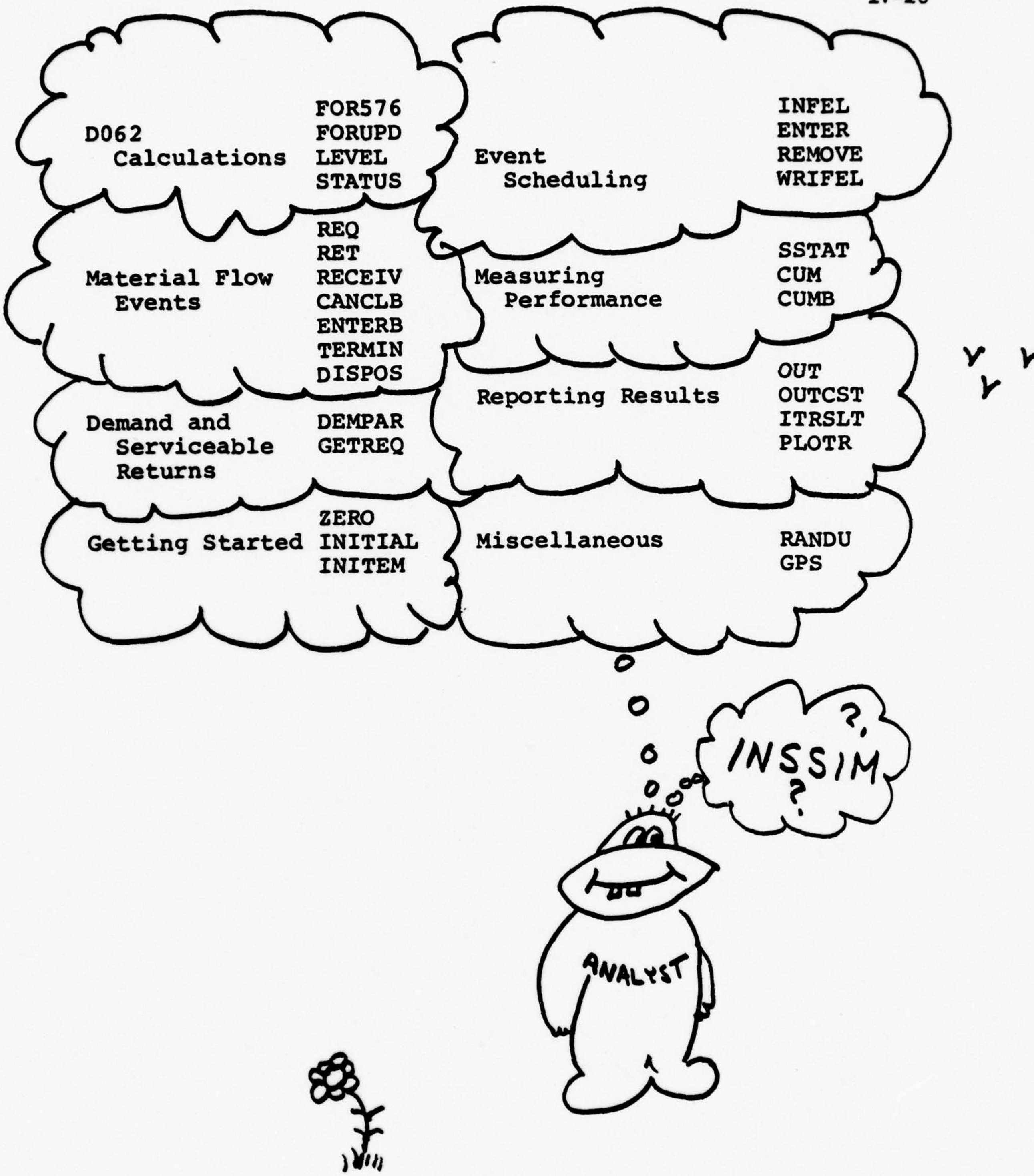


Figure IV-6. Major INSSIM Subroutines

Subroutine	Function	Description
STATUS	Compares available assets to the respective control levels, and initiates appropriate actions	This routine simulates the management action portion of D062 system.

Each of the above routines contain logic describing the computational formulas and management policies currently used in the D062 system. In addition, these routines also possess logic describing several forecasting and inventory control procedures that have been suggested as alternatives to current methods. These alternate procedures may be simulated by changing one of the eight parameter cards that specify the characteristics of a given simulation run. Hence, a number of alternate inventory management proposals may be evaluated by simply changing the input specifications to INSSIM.

In addition to the D062-related subroutines described above, several other INSSIM routines are used to describe significant events in the flows of EOQ items. These events and their corresponding subroutines are as follows:

Routine	Event
REQ	A requisition is received at the depot. This represents a demand for a specific number of units of a particular EOQ item. If possible, the requisition is filled immediately; otherwise, the requisition is backordered until a replenishment order is received.
RET	A number of serviceable units are returned to the supply system.
RECEIV	A replenishment order is received by the supply system.
CANCLB	A customer with an outstanding backorder cancels the requisition.
ENTERB	Record the current requisition as a back-order, and insert it into the backorder file.
TERMIN	Action is taken to stop a replenishment order that has not yet been received. This event is initiated whenever an item's asset position exceeds the termination level, and a replenishment order is still being processed.
DISPOS	Assets in long supply are disposed of.

(Note: The routines CANCLB, TERMIN, and DISPOS were not used in the current study.)

Each of the above routines perform bookkeeping operations that update the status of on hand and on order stocks. These routines also post activity statistics that record inventory system performance.

The methods for describing the demand generation process is a critical element in any inventory simulation. IN INSSIM, the demand generation process is derived from demand

and serviceable return histories for actual Air Force items. Specific computational details are handled by subroutines DEMPAR and GETREQ.

As we shall see below, a major input to our study is the actual demand histories for a sample of EOQ items from the EOQ data bank. This input defines the actual quarterly demands and serviceable returns for each item for the period FY 71 through FY 76, a total of 20 quarters worth of data. In our study, the first eight quarters of data were used to initialize the history files needed in the D062 usage rate calculations. The remaining 12 quarters of data were used to simulate demands in the inventory system.

The demand generation process is constructed so that within a particular quarter, the number of units of demand and the number of serviceable returns simulated exactly equals the actual values from the EOQ data bank. Within a given quarter, specific requisitions are generated that have the same statistical characteristics as current USAF items. Specifically, requisition sizes are generated according to the probability distributions presented in Figure II-2.

As shown in Figure IV-6, a number of other routines are also used in INSSIM. These routines are required to initialize the simulation model, to collect and summarize

performance statistics, and to assist in event scheduling and other bookkeeping tasks. Rather than discuss these routines individually, the following sections describe the ultimate results of these routines upon INSSIM input requirements and output products.

INSSIM Inputs and Outputs

Figure IV-7 illustrates the major data flows of the Inventory System Simulator. Run specifications are input from File 05 in card format. This input specifies the inventory policy options that are to be simulated in the current run, as well as significant parameters (e.g., holding cost, ordering costs, and bounds on EOQ's and safety stocks) required by these policies. Other input cards specify the output options to be employed, and the size of the simulation run (e.g., number of items to be simulated, time duration for the study, etc.). A printback of the run specifications for a 100 item, 12 quarter simulation run is shown in Figure IV-8.*

As shown in Figure IV-7, item demand and cost data is input through File 07. This file provides item information extracted from the EOQ Data Bank. This file contains data

* A detailed discussion of this output product is presented in Volume I, Chapter VII of Reference 2.

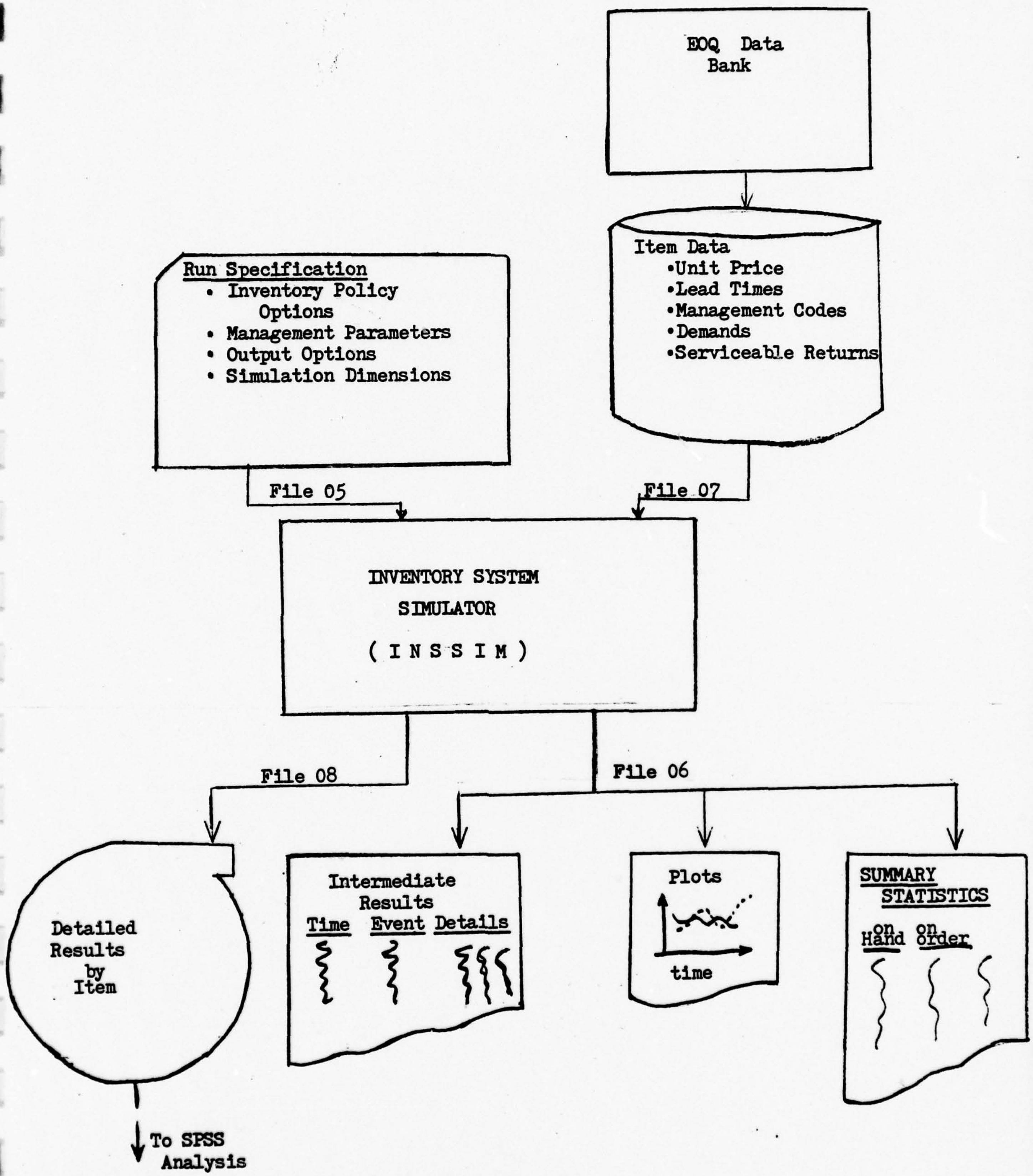


Figure IV-7. INSSIM Input/Output Relationships.

```
                    INVENTORY SYSTEM SIMULATOR
                          RUN PARAMETERS

(C1)  RUN-ID.     3  TITLE. 100 ITEM SIMULATION WITH ZERO SS

(C2)  OUTPUT CONTROLS...(NOTE. 1=YES)
ITWRT          IT.WRITE=  0
IOUT           SUMMARY =  1
IGRAPH         GRAPHS  =  1
IPUNCH         PUNCH   =  1

(C3)  DEBUG FLAGS
               IDBUG =  0
               IEBUG =  1
               IFBUG =  0
               IGBUG =  0
               IHBUG =  0

ITRACE         START TRACE AT      0   FOR ITEM 1
ISTRAC         STOP TRACE AT     0

(C4)  ITEM INPUT FILES
INLU           FILE =  7
INTYPE         TYPE =  2   (1 = BCD; 2 = BINARY)
NDEM           QTRS = 24

(C5)  MANAGEMENT METHODS TO BE USED
ICDFOR         FORECAST FORMULA   =  1
ICDSIG         STD DEV   FORMULA  =  1
ICDEOQ         EOQ       FORMULA  =  2
ICDSL          SAFETY STK    CODE =  3
ICDSLL         SAFETY LIMIT CODE  =  1
ICDBG          BUDGET GUIDE CODE  =  1
ICDSR          RETURNS CODE       =  1

(C6)  MANAGEMENT PARAMETERS
EOQMIN         MIN EOQ(MNTHS)               6.0000
EOQMAX         MAX EOQ(MNTHS)              36.0000
SLMIN          MIN SAFETY LV(MNTHS)         0.
SLMAX          MAX SAFETY LV(MNTHS)         0.

(C7)  SYSTEM PARAMETERS
COSHLD         HOLDING COST/$-INV           0.2000
CSHORT         SHORTAGE COST               10.0000    31.6000   100.0000

COSORD(1)      SMALL ORDER COST           269.8700
COSORD(2)      LARGE ORDER COST           460.2700

CSTORK         COST BREAK-POINT         10000.0000

(C8)  SIMULATION SIZE
NRUN           NUMBER OF RUNS               1
INQTR          NUMBER OF QUARTERS          12
NREPL          NO. OF REPLICATIONS        100
NITEM          NO. OF ITEMS/REPL            1
```

Figure IV-8. Specifications for a 100 Item 12 Quarter Simulation Run.

defining the unit price, lead times, management codes, demands and serviceable returns associated with each item to be simulated.

Output products produced by INNSSIM are routed to files 06 and 08. File 08 is a magnetic tape file. It contains details by quarter on the performance of each item simulated. This file is designed for subsequent statistical analysis of the simulation results using the SPSS Statistical Package.

File 06 is routed to the printer. If all output options are requested, this file will contain a print-back of all input data, an event-by-event description of the simulation process, and detailed plots and statistical summaries of simulation results.

Figures IV-9 through IV-13 illustrate some of the performance summaries produced by INSSIM.* For example, Figures IV-9 provides statistics describing the number of units on hand and on order at the end of each simulated quarter, as well as counts of the number of units received from vendor shipments and serviceable returns. Figure IV-10 presents similar counts of the number of expediting, rationing, disposal, and termination actions taken in the simulation, while Figure IV-11 presents data describing average inventories and fill and backorder rates. Finally, Figure IV-12

* These figures show aggregate statistics obtained by summing over all items simulated.

PERFORMANCE STATISTICS

... UNITS ...

PERIOD	(1) INVENTORY ON HAND	(2) INVENTORY ON ORDER	(3) RECEIPTS	(4) RETURNS	(5) TOTAL SHIPMENTS	(6)** PRIORITY 1 TOTAL SHIPMENTS	(7) ORDERS PLACED	(8) TOTAL REQUISITIONS	(9) REQS CANCELED	(10)** PRIORITY 1 REQUISITIONS
BOP	103865	0								
1	93126	28931	0	3121	13860	13860	28931	14959	0	14959
2	88418	33103	8455	3483	16646	16646	12627	16213	0	16213
3	90113	29159	17413	2533	18251	18251	13469	19266	0	19266
4	85072	28422	13150	2472	20663	20663	12413	21627	0	21627
5	81929	30703	12957	931	17031	17031	15238	15914	0	15914
6	81757	33183	13535	909	14616	14616	16015	14591	0	14591
7	84967	21260	16181	613	13584	13584	4258	13473	0	13473
8	88260	22787	16672	799	14178	14178	18199	13328	0	13328
9	77434	32058	5802	1199	17827	17827	15073	17791	0	17791
10	74589	39341	7981	937	11763	11763	15264	13507	0	13507
11	81409	27490	19248	899	13327	13327	7397	12947	0	12947
12	87396	15637	20688	759	15460	15460	8835	14148	0	14148
TOTALS	1014470	342074	152082	18655	187206	187206	167719	187764	0	187764
AVE/YR	338157.	114025.	50694.	6218.	62402.	62402.	55906.	62588.	0.	62588.

(1) Inventory on hand at the end of the quarter.

(2) Total quantity on order at the end of the quarter.

(3) Receipts of the replenishment orders.

(4) Serviceable returns from customers of the supply system.

(5) Total shipments to fill new customer requisitions or to fill backorders.

(6) Total shipments to fill priority 1 requisitions or backorders.

(7) Total replenishment orders initiated.

(8) Total requisitions received.

(9) Total outstanding backorders that are canceled by the customer.

(10) Total priority 1 requisitions received.

Figure IV-9. Receipts, returns, requisitions, and end-of-period measures.

** Note: In the current study, all demands are priority 1.

PERFORMANCE STATISTICS

... UNITS ...

	(11)	(12)	(13)	(14)
PERIOD	EXPEDITES	RATIONING ACTIONS	DISPOSALS	TERMINATIONS
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
TOTALS	0	0	0	0
AVE/YR	0.	0.	0.	0.

(11) Total expediting actions initiated.

(12) Total of all rationing actions within the period.

(13) Total disposals within the period.

(14) Total terminations; i.e., totals for all replenishment orders that are cancelled or reduced when the inventory position exceeds the termination level.

Figure-10. Expediting, rationing, disposal, and termination measures.

PERFORMANCE STATISTICS

UNITS

PERIOD	(15) TOTAL BACKORDERS	(16)** PRIORITY 1 BACKORDERS	(17) TOTAL BACKORDER weeks	(18)** PRIORITY 1 BACKORDER weeks	(19) INVENTORY weeks	(20) TOTAL FILLS	(21)** PRIORITY 1 FILLS	(22) TOT FILLS /TOT RFQ	(23)** PRI 1 FILLS /PRI 1 RECS
1	1099	1099	3127	3127	1186380	13828	13828	0.92	0.92
2	666	666	13580	13580	1076624	15418	15418	0.95	0.95
3	1681	1681	12507	12507	1064541	17749	17749	0.92	0.92
4	2645	2645	17097	17097	1074017	18765	18765	0.87	0.87
5	1528	1528	22900	22900	1006883	13000	13000	0.82	0.82
6	1503	1503	18692	18692	991353	13577	13577	0.93	0.93
7	1392	1392	20116	20116	1045332	12524	12524	0.93	0.93
8	542	542	12731	12731	1036357	12147	12147	0.91	0.91
9	506	506	3780	3780	987006	16738	16738	0.94	0.94
10	2250	2250	16921	16921	882825	11057	11057	0.82	0.82
11	1670	1870	25436	25436	955096	9569	9569	0.74	0.74
12	558	558	13463	13463	1015204	12686	12686	0.90	0.90
TOTALS	16240	16240	180350	180350	12321618	167058	167058		
AVE/YR	5413.	5413.	60117.	60117.	4107206.	55686.	55686.	0.890	0.890

(15) Total backorders outstanding at the end of the period.

(16) Total priority 1 backorders outstanding at the end of the period.

(17) Total backorder-weeks observed during the period. For example,,if one item has a single requisition for 12 units in a backorder status for 3 weeks, there are 3 requisition-weeks of backorders, and 3 X 12 = 36 unit-weeks of backorders for that item.

(18) Total back-order weeks observed for priority 1.

(19) Total number of inventory-weeks observed. If there are 15 units on hand for the first 8 weeks of a quarter, and 6 units on hand for the remaining 4 weeks, a total of (15 X 8 + 6 X 4)= 144 inventory unit-weeks were observed in the period.

(20) Totals for all requisitions that were filled "off-the-shelf",i.e., that were filled without backordering.

(21) Total fills for priority 1 requisitions.

(22) Fill rate for the period.

(23) Priority 1 fill rate.

** Note: In this study, all demands are priority 1.

Figure IV-11. Fill rate and backorder measures.

***** ORDER COST STATISTICS *****

TIME	SMALL ORDERS			LARGE ORDERS			TOTAL ORDERS		
COST TO ORDER		269.87			460.27				
	NUMBER	UNITS	DOLLARS	NUMBER	UNITS	DOLLARS	NUMBER	UNITS	DOLLARS
BOP	0	0	0	0	0	0	0	0	0
1	22	27366	97649	2	1565	30896	24	28931	128545
2	15	11533	64864	1	1094	14222	16	12627	79086
3	10	13469	32130	0	0	0	10	13469	32130
4	9	9434	37661	3	2979	38727	12	12413	76388
5	14	15111	57250	1	127	13843	15	15238	71093
6	15	16015	50355	0	0	0	15	16015	50355
7	4	4258	13429	0	0	0	4	4258	13429
8	13	16996	43146	2	1203	24504	15	18199	67650
9	15	13492	55123	1	1581	20553	16	15073	75676
10	13	15130	50260	1	134	14606	14	15264	64866
11	10	6000	42077	2	1397	25567	12	7397	67644
12	9	7605	28807	2	1230	24774	11	8835	53581
TOTALS	149	156409	572751	15.	11310	207692	164	167719	780443

SMALL ORDER COST 0.40210630E 05 PLUS LARGE ORDER COST 0.69040500E 04 EQUALS TOTAL ORDER COST 0.47114680E 05

FOQ BUY DOLLAR BREAK POINT 10000.00

AVERAGE COST/ITEM/YR

SMALL ORDER COST 0.13292280E 37 PLUS LARGE ORDER COST 0.13292280E 37 EQUALS TOTAL ORDER COST 0.26584560E 37

Figure IV-12. Order Cost Summaries.

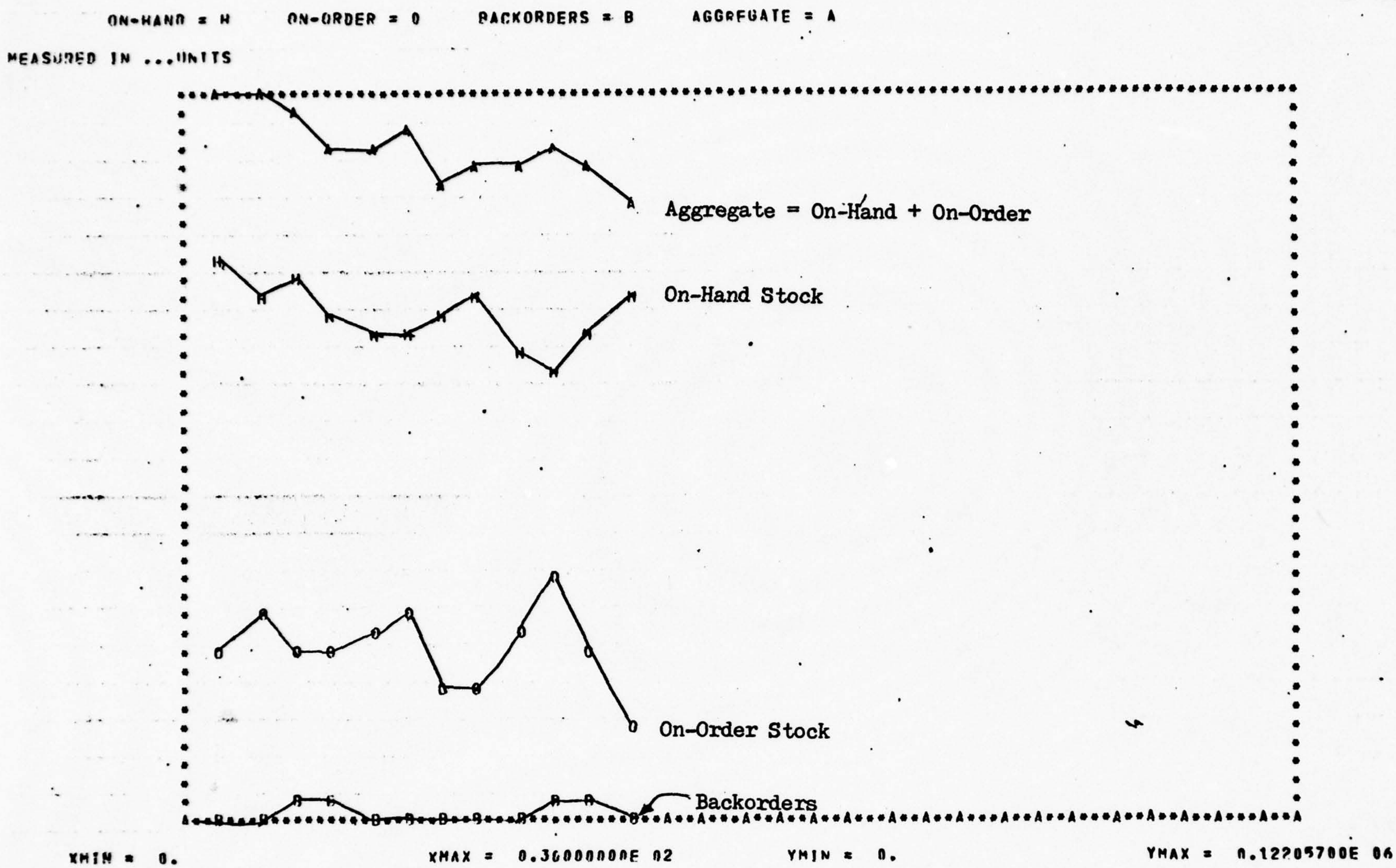


Figure IV-13. On-hand and on-order assets and backorders verses time.

summarizes ordering actions using large and small purchasing methods, and Figure IV-13 plots on hand stocks, on order stocks and backorders as a function of time.

In INSSIM, all statistics are accumulated according to three different measures; they are:

a. The number of distinct federal stock numbers or distinct actions associated with the current event.

b. The quantity of units associated with the event.

c. The dollar value of all units associated with the event.

For example, suppose a replenishment order for 12 units of a $10 item is placed. In this case, INSSIM records that there was one order action, 12 units were ordered, and $120 was the value of the order. The results presented in Figures IV-9 through IV-13 are all reported in terms of unit counts. However, INSSIM also produces similar tables summarizing the action and dollar counts recorded in the simulation.

This concludes our discussion of the Inventory System Simulator. In the next section, we will discuss how this model was used to evaluate the relative cost-effectiveness of the alternate PT-formulas.

Section V. Simulation Results

This section presents the results of a simulation study to evaluate the relative cost-effectiveness of the three alternate versions of the Presutti-Trepp formulas defined in Appendix A. To perform this study, it is necessary to accummulate, screen, and reformat available EOQ Data Bank records into a form suitable for use in the simulation model. To do this required development of several computer programs to accomplish the extraction task.

Figure V-1 illustrates the major steps involved in the data extraction process. As illustrated in the figure, three major processing streams, or branches, were implemented. Branch A illustrates those processing steps needed to develop the frequency distributions for average requisition size. This step also evaluated other statistical characteristics of items currently in AFLC inventories. Branch A concentrates attention on the most recent available data, namely, data from the period July - September 1976. The major results of these analyses are presented in Section II. Processing branches B and C relate to the construction of physical records appropriate for input to the Inventory System Simulator, and to the statistical analysis, sampling, and simulation activities which utilize these records.

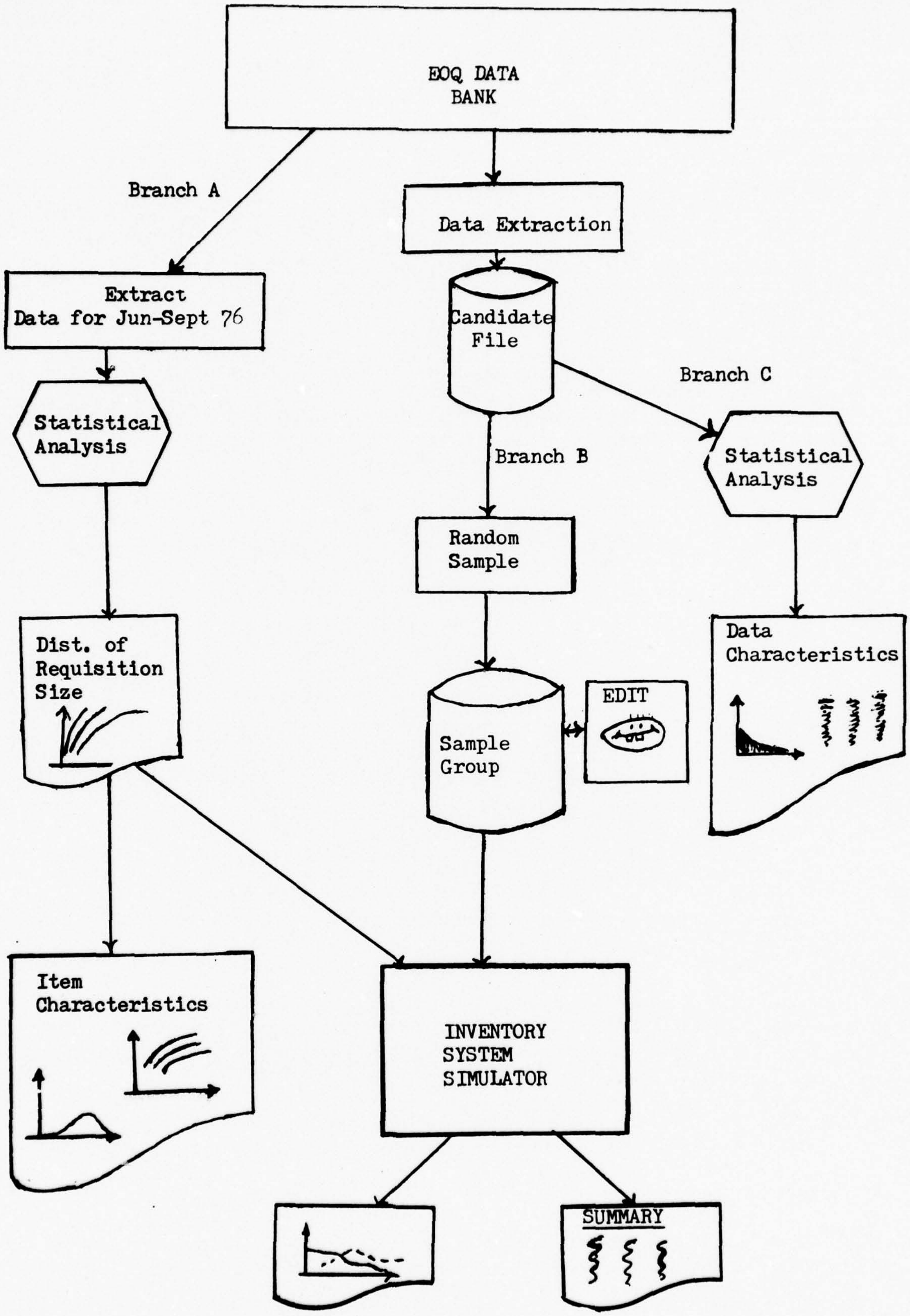


Figure V-1. Major Steps in the Data Extraction Process

INSSIM assumes that all input information for a given item will be contained on adjacent physical input records. Unfortunately, the EOQ Data Bank records are not arranged in this order. Rather, information required to simulate a given item is located on several different EOQ Data Bank files. For example, there is a separate history file for each fiscal year. Also, unit price and inventory management codes are contained on file 01, while file 02 contains on-hand and on-order stock status. This latter data is required to initialize the simulation. Hence, a major task in this project was to extract and reformat all relevant data records. We called the result of this extraction process the "Candidate" file.

In building the Candidate file, we deleted all items that were not managed according to the EOQ methodology, as well as items with incomplete demand histories. Specifically, an item was <u>excluded</u> from the Candidate file if:

a. The item had no demand in the FY 71 to FY 72 time period. It was essential that an item have at least some demand in this period to permit computations of reorder points and reorder levels at the beginning of the simulation.

b. The item was coded as Type Computation = C. This computation code is applied to items with a short shelf or program life, items subject to a calendar time change, and to other items for which the EOQ assumptions are not valid or for which requirements cannot be related to past demand.

c. The item possessed a Special Code of

C = contingency	X = obsolete
D = disposal	N = new item
I = insurance	U = use till exhausted
M = I/S breakdown	

d. The demand records for the item for the FY 71 through FY 76 period were incomplete. This would happen, for example, if an item was phased out of the inventory, or was transferred to another ALC.

Approximately 80,000 records were in the Sacramento ALC FY 71 History File. A total of 12,339 of these items passed the above criteria and were written to the Candidate file. Each of these items were candidates for inclusion in our simulation runs. Since there were many more candidate items than needed for our study, random sampling was performed to provide inputs to the simulation model.

Samples by dollar value of demand

Because of the high budgetary impact associated with items that have high annual values of demand, we initially selected three samples for simulation analysis based upon the annual dollar demand rate of the items in the FY 71 - FY 72 period. In our study, three classes of items were identified:

CLASS	DEFINITION
HI	Demand exceeds $5,000 per year
MID	Demand is between $500 per year and $5,000 per year
LO	Demand is less than $500 per year.

Our sampling procedure consisted of two steps. First, approximately 120 items were randomly selected from the Candidate File for each of the three classes defined above. Items in these samples were then carefully edited. The objective of the

editing process was to eliminate any "weird" items (e.g., items with key-punch errors) in the sample that would bias the simulation results. After editing, 100 items from each class were selected for detailed analysis.

The Simulation scenario

Once the item samples were constructed, as number of simulation runs were performed to evaluate the operating characteristics of each of the alternate PT-formulas in managing each of the item samples. In all runs, the following parameters were used:

Holding cost = 20% of the item unit price per year

$$\text{Order cost} = \begin{cases} \$269.87 & \text{if order was for less than } \$10{,}000 \\ \$460.27 & \text{otherwise} \end{cases}$$

Maximum EOQ = 36 months of supply

Minimum EOQ = 6 months of supply

These are the parameter values currently used in the DO62 system.

As noted above, 20 quarters of demand history from the FY 71 through FY 75 period were available for our study. The oldest 8 quarters of this data were used to initialize the DO62 forecast calculations. The remaining 12 quarters of data were used to simulate demand and serviceable returns for a three-year period.

To initialize stock status, we set the initial on-hand assets for each item equal to the total assets on hand for that

item as of June 1974, the earliest period for which stock status records were available. We also needed to initialize on-order stocks. Rather than develop an elaborate method for simulating the arrival time of on-order stocks, we simply increased the initial on-hand assets by the number of units on order as of June 1974. Hence, initial on-hand stocks in our simulation runs should be higher than would have been the case in practice; however, this effect would be the same for each formula set, and thus should not effect the relative performance of these formulas.

Our simulation strategy was based on an analysis of the respective safety level calculations. Observe that the safety level of each of the formula sets in Appendix A is given by Safety Level = $K\sigma$, where:

$$K = .707 \ln \left[\frac{\lambda}{2\,ac} \cdot \frac{1}{Z} \cdot \frac{\sigma\,(1 - \exp(-\sqrt{2}\,Q/\sigma))}{\sqrt{2}\,Q} \right]$$

K = safety factor
λ = implied shortage factor
a = holding cost per dollar of inventory per year
c = unit price
σ = standard deviation of demand in the lead time
Q = order quantity
Z = essentiality factor

The safety level is then bounded to be no less than zero and no more than the lessor of 3σ or the expected number of demands in the lead time.

As noted above, the holding cost (a) was set to 20% in all simulation runs. The values of c, Q, and σ were determined from the particular demand and cost characteristics of each item being

simulated. The value of Z, the essentiality factor, is determined by the particular formula set being evaluated. Specifically,

$$\text{for Formula Set} \quad \begin{cases} \text{I} & Z = \sqrt{R} \\ \text{II} & Z = 1 \\ \text{III} & Z = R \end{cases}$$

where R denotes the average requisition size, and where the formula sets I, II, and III are identified in detail in Appendix A.

The implied shortage cost, λ , serves a special role in the safety level calculation. As we saw in Section II, if the cost of shortage λ is small relative to the cost of holding inventory (ac), the optimum safety level will also be small. Hence, a small value of λ will produce a small level of safety stock investment, and a relatively high number of back orders. On the other hand, if the cost per unit short is high, a higher safety level will be computed. This produces higher inventory investment, but it also reduces the number of back orders expected. Thus, the parameter λ is a "management control knob" that controls the relationship between safety stock investments and system back orders.

The parameter λ directly controls the safety level and the reorder point for every inventory item. The larger the value of λ , the larger the computed safety level and reorder point will be. But the reorder point controls the amount of buy notices that will be triggered in a given time period. Hence, λ may be used to control the amount of money spent in a given

fiscal period. Higher values of λ will lead to higher reorder points, and thus lead to higher buys in a given period, while lower values of λ lead to lower expenditure levels.

Because of the control knob effect of the parameter λ upon both investment and back order levels, each formula set was simulated eight times. The parameter λ was set to the values of $10, $31, $100, $316, $600, $1000, $3,162, and $10,000, respectively, in each of these runs. Hence, since there are three formula sets, a total of 8 x 3 = 24 runs were required for each item sample to be simulated. To provide a common comparison point, one additional run was made for each item sample. In this run, the safety level was set to zero for all items simulated.

Results

Table V-1 summarizes the results for 25 runs using the HI group of items, i.e., the 100 item sample with FY 71 - FY 72 demands of $5,000 or more per year. In each of these runs, a total of 3,616 requisitions were submitted for a total of 8,489 units. Since we simulated each of the 100 items for a total of 12 quarters, an "average" item had 36 requisitions for 84 units during the 3 year simulation period. The average requisition size was thus 84 ÷ 36 = 2.3 units per requisition. Of course, no single item is average; as we saw in Section II, there is generally great variability among items.

Table V-1 summarizes four of the performance measures associated with each simulation run. The column "REQ-BO weeks"

presents the number of weeks that requisitions were back ordered during the 3 years simulated. For example, if two requisitions were back ordered for three weeks during a particular quarter in the simulation, a total of 3 X 2 = 6 requisition weeks would be recorded. In Run #1, there was a total of 8,043 requisition weeks of back orders in the three year simulation period. This is an average of 2,681 requisition weeks of back orders each year. This average value is shown in the first line of Table V-1. Similarly, if the two requisitions were each for 20 units, a total of 2 X 3 X 20 = 120 unit weeks of back orders would be recorded. In Run #1, there was a total of 68,733 unit weeks of back orders observed, an annual average of 22,911 unit weeks of back orders for the collection of 100 items. This average annual value is shown in the second column of Table V-1. The third column of this table displays the average annual buy dollars associated with each run. For example, for Run #1, an average of $897,000 worth of orders were placed each year of the simulation. Finally, the fourth column presents the average annual cost of holding inventories for the 100 item group. Since holding cost equals 20% of the average inventory investment, dividing each entry in column 4 by .2 yields the average investment observed in each simulation run.

So far, we have discussed only the first line of Table V-1. This line summarizes the results of Run No. 1, the run in which the safety level was set to zero for each item simulated. The remaining lines in Table V-1 summarize the results of the other 24 simulation runs performed for the HI item group.

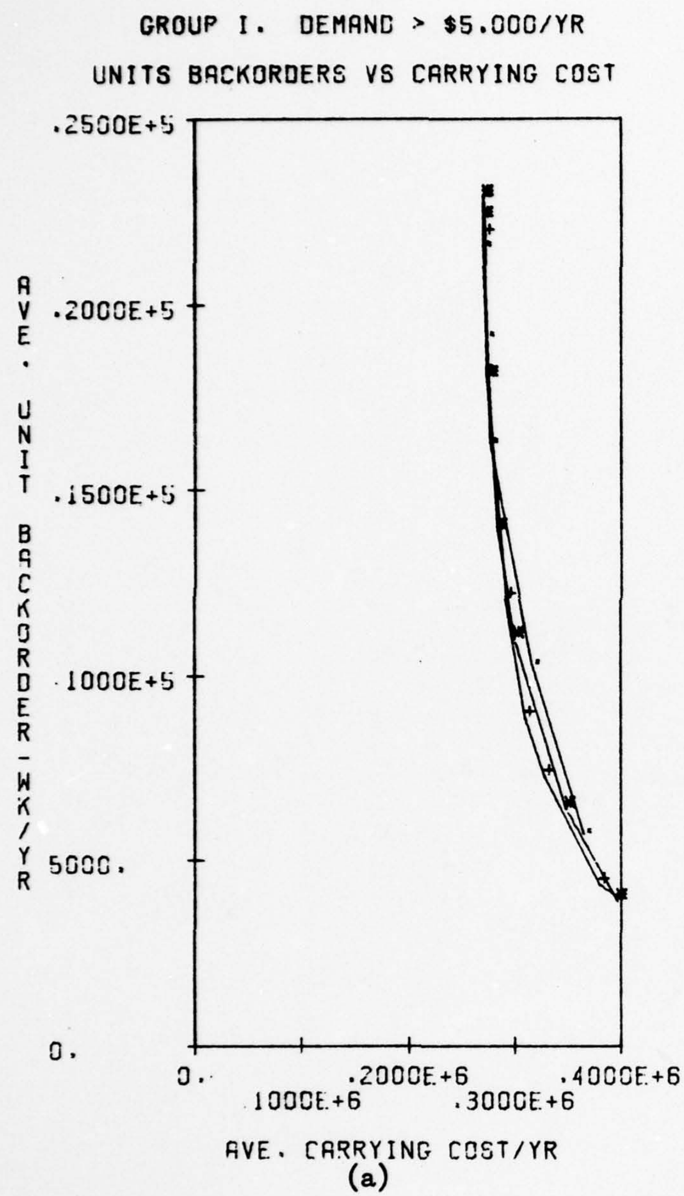
Table V-1
Average Annual Results for 100 Items
in HI Class

RUN NO.	FORMULA	λ	① REQ-BO WEEKS	② UNIT BO WEEKS	③ BUY DOLLARS	④ HOLDING COST
	Zero Safety Level					
1			2,681	22,911	897,020	270,908
	$Z = 1$					
2		10.	2,681	22,911	897,020	270,908
3		31.	2,645	21,824	897,545	271,448
4		100.	2,408	18,075	903,792	274,677
5		316.	1,809	12,056	958,202	291,316
6		600.	1,269	8,823	1,025,940	310,365
7		1,000.	1,034	7,258	1,084,167	328,984
8		3,162.	540	4,328	1,274,385	380,304
9		10,000.	407	3,505	1,428,238	426,112
	$Z = \sqrt{R}$					
10		10.	2,681	22,911	897,020	270,908
11		31.	2,681	22,911	897,020	270,908
12		100.	2,671	22,308	897,309	271,089
13		316.	2,344	18,033	906,262	274,690
14		600.	1,880	13,916	934,000	284,133
15		1,000.	1,515	10,974	988,703	300,548
16		3,162.	814	6,405	1,143,687	347,859
17		10,000.	476	3,933	1,331,825	397,255
	$Z = R$					
18		10.	2,681	22,911	897,020	270,908
19		31.	2,681	22,911	897,020	270,908
20		100.	2,681	22,911	897,020	270,908
21		316.	2,615	21,545	897,020	270,972
22		600.	2,363	19,105	901,166	273,047
23		1,000.	2,120	16,230	926,473	277,112
24		3,162.	1,260	10,278	1,025,331	316,371
25		10,000.	676	5,744	1,198,788	364,858

As noted above, eight simulation runs were performed for each formula set to be evaluated. Simulation runs 2 through 9 all used the Z=1 formula, but higher and higher values of the control parmenter λ were used in each successive run. Similarly, Runs 10 through 17 and Runs 18 through 25 were used to evaluate the $Z=\sqrt{R}$ and Z=R formulas, respectively. For each group of runs, observe that as λ increases, both the average buy dollars and the average inventory carrying cost increase. On the other hand, the higher costs are also accompanied by reduced levels of requisition and unit backorders. Observe that a similar pattern also holds for each of the other safety level formulas.

Note that for each formula set, the results for the run in which λ = $10 are identical to results for the Zero Safety Level run. This is because a low value of λ yields a safety level of zero in each of these formulas. Once λ becomes large enough, however, positive safety levels are computed, and improved operating results are observed.

The above patterns may be seen more clearly in graphs of the Table V-1 results. These graphs are presented in Figure V-2 through V-5. In Figure V-2(a), we have plotted the average unit weeks of backorders verses the average annual carrying cost, while Figure (V-2(b) plots requisition-backorders verses annual carrying cost. In these and the following figures, a "+" denotes runs in which Z=1, a "*" denotes runs with $Z=\sqrt{R}$, and a "." denotes



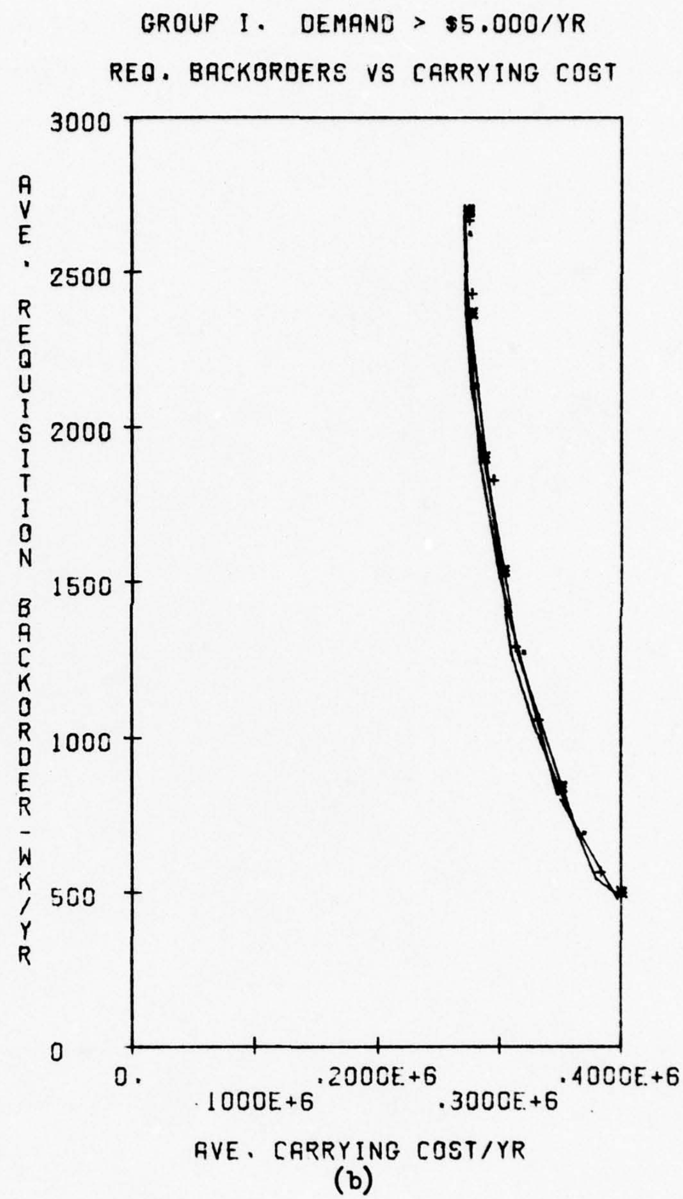
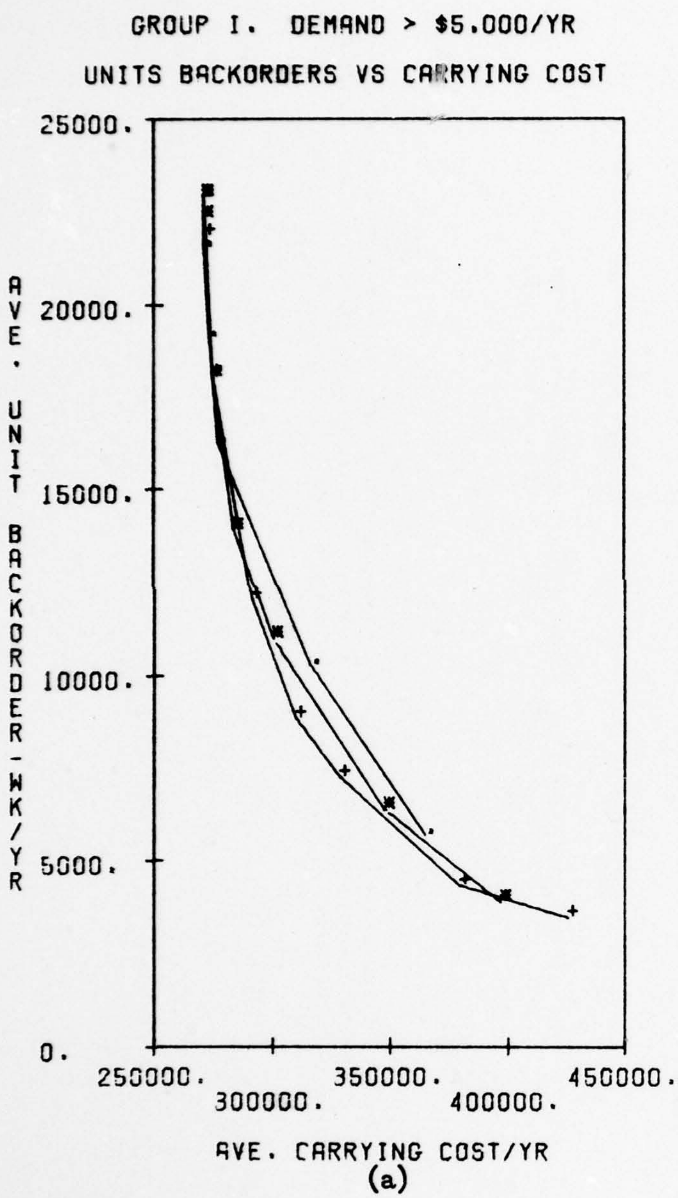


Figure V-2. Backorders verses Annual Carrying Cost for HI Group.



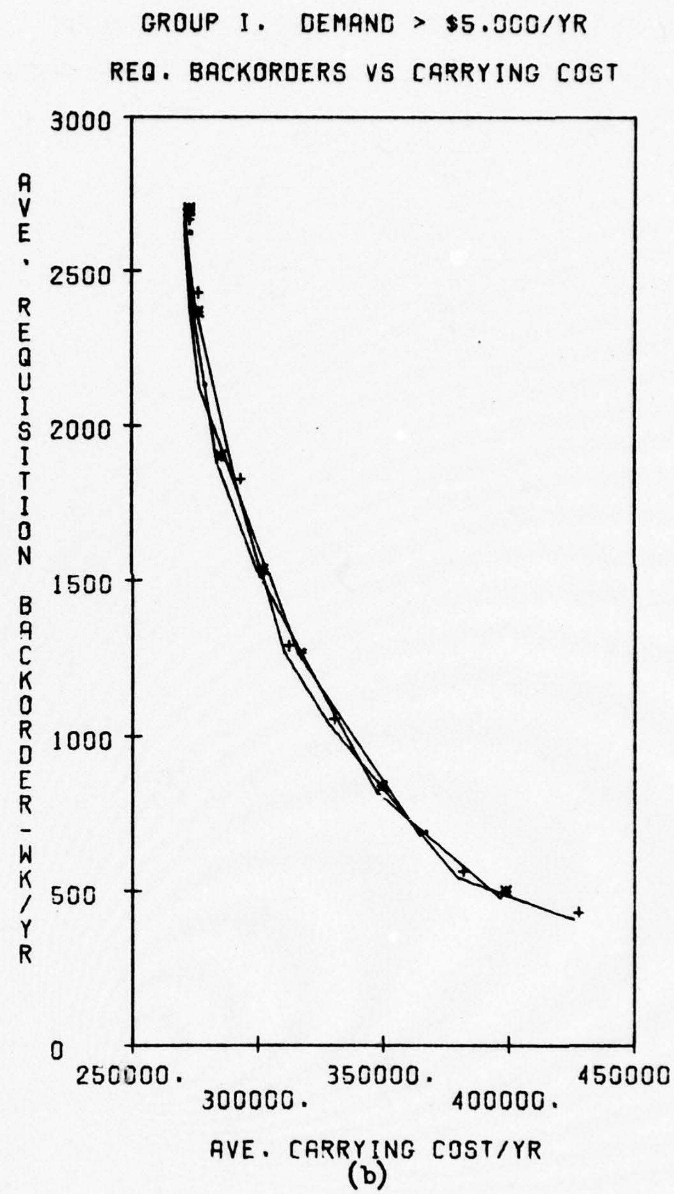
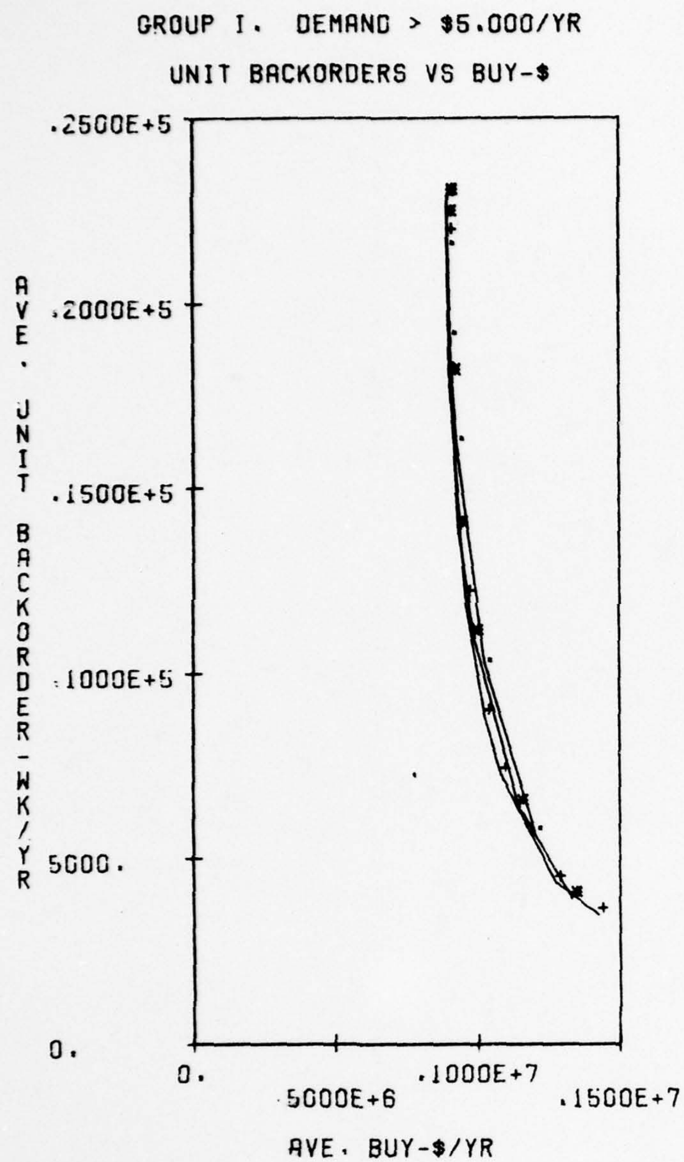
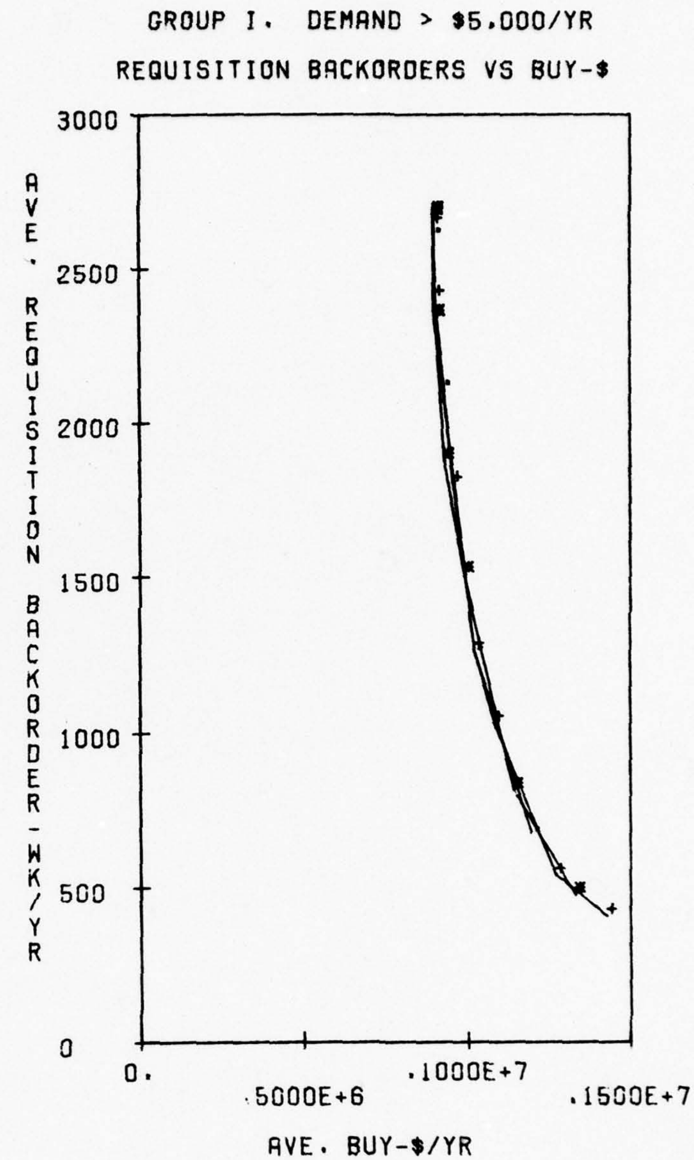


Figure V-3. "Magnified" plot of Backorder/Carrying Cost Curve for HI Group.



(a)



(b)

Figure V-4. Backorders verses Annual Buy Dollar for HI Group.

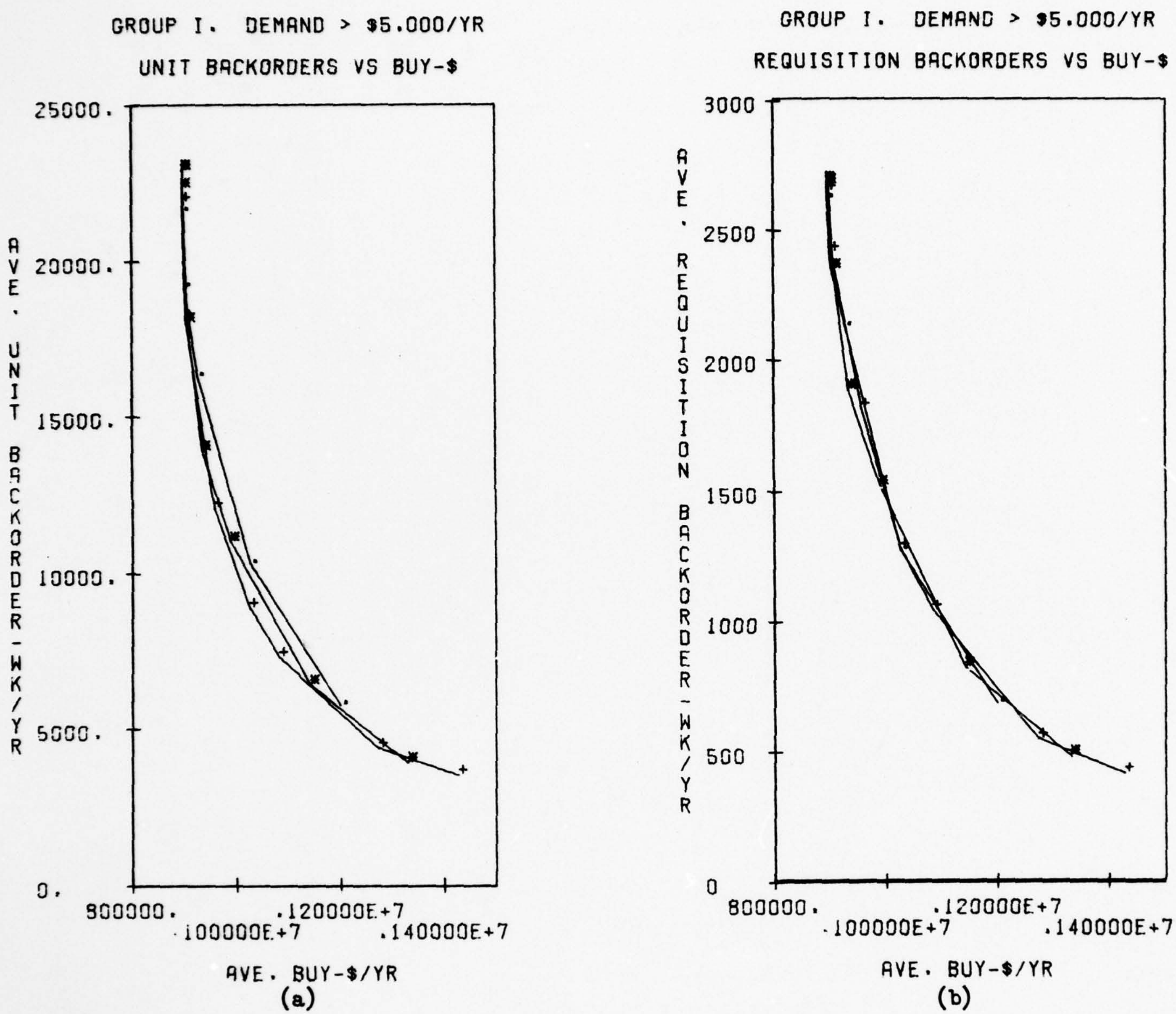


Figure V-5. Backorders verses Buy Dollars, HI Group.

runs with runs with Z=R, where R denotes the average requisition size. Observe that all three curves are very similar in both plots. Note, however, that in Figure V-2(a), the Z=1 (+) curve is consistently below and to the left of the other two curves. Also, note that the $Z=\sqrt{R}$ (*) curve lies consistently between the other two curves. These relationships are even more apparent in Figure V-3(a), where we have magnified the three curves by changing the changing the scale of the plots.

Curves V-2(a) and V-3(a) indicate that the Z=1 formula is more cost-effective with respect to unit backorders than the other two formulas. That is, a given level of unit backorders was achieved with the Z=1 formula at a lower annual carrying than were observed for the other two formula sets. Conversely, for a given value of annual carrying cost (or a given value of inventory investment), the Z=1 formula produced a lower level of backorders. Similarly, the $Z=\sqrt{R}$ formula performed consistently better than the Z=R curve with respect to unit backorders.

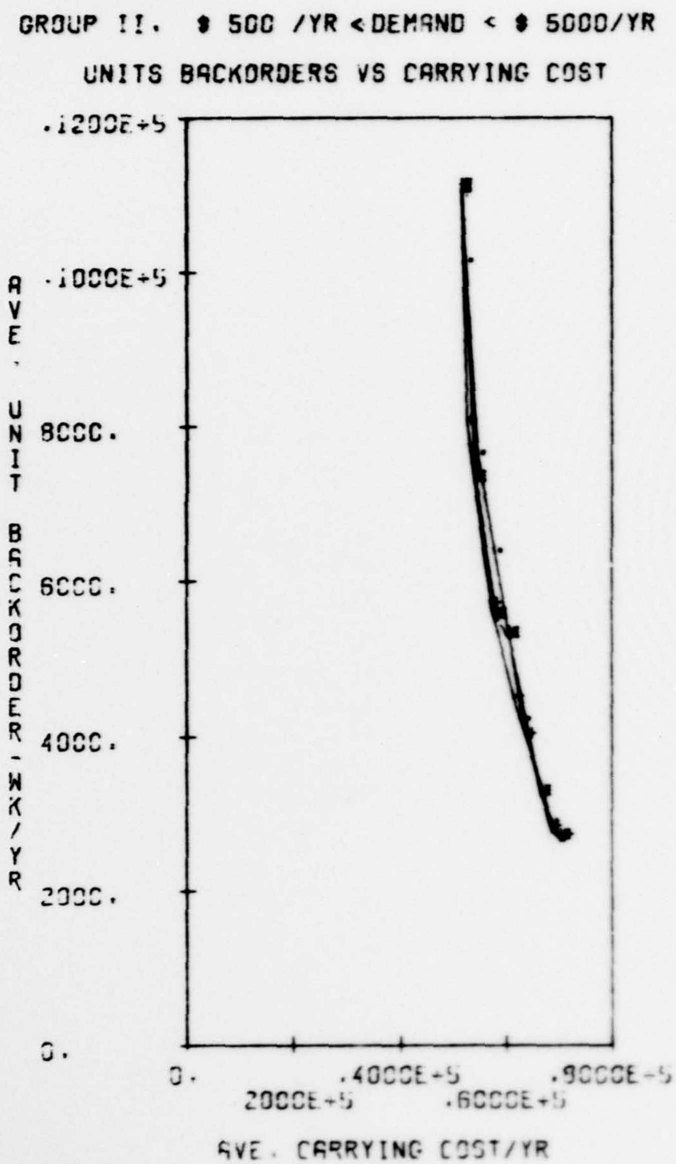
The above results apply to the relationship of annual carrying costs and unit-weeks of backorders. If backorders are measured in terms of requisition-weeks, however, there appears to be no clearly superior formula. As shown in Figure V-2(b) and V-3(b), all three curves lie approximately along the same line. None of the curves are consistently better than the others.

Similar results are obtained if unit-backorders and requisition-backorders are ploted verses average annual buy dollars. These results are plotted in Figures V-4 and V-5. Figure V-4 is plotted with a "0, 0" origin, whiles Figure V-5 is a "magnified" version of the same data. Observe that the Z=1 (+) curve is consistently better than the other two formulas when unit-backorders are plotted verses average annual buy dollars. On the other hand, there appears to be no significant differences among the curves when requisition backorders are plotted verses buy dollars.

We obtained similar results for the MID volume item sample, i.e. the sample with FY71-FY72 demand in the \$500-\$5000/year range. These curves are presented in Figures V-6 through V-9. Observe that the Z=1 curve continues to dominate the other curves when unit-backorders are plotted against either annual carrying costs or annual buy dollars.

Now look at Figures V-6(b), V-7(b), V-8(b), and V-9(b). Observe that the Z=1 (+) and the $Z=\sqrt{R}$ (*) curves are very similar, but that both are better than the Z=R (.) curve.

Results for the LO volume group of items are presented in Figures V-11 through V-13. These 100 items had demands of \$500/year or less in the FY71-FY72 period. They also tended to have very low levels of activity, which produced



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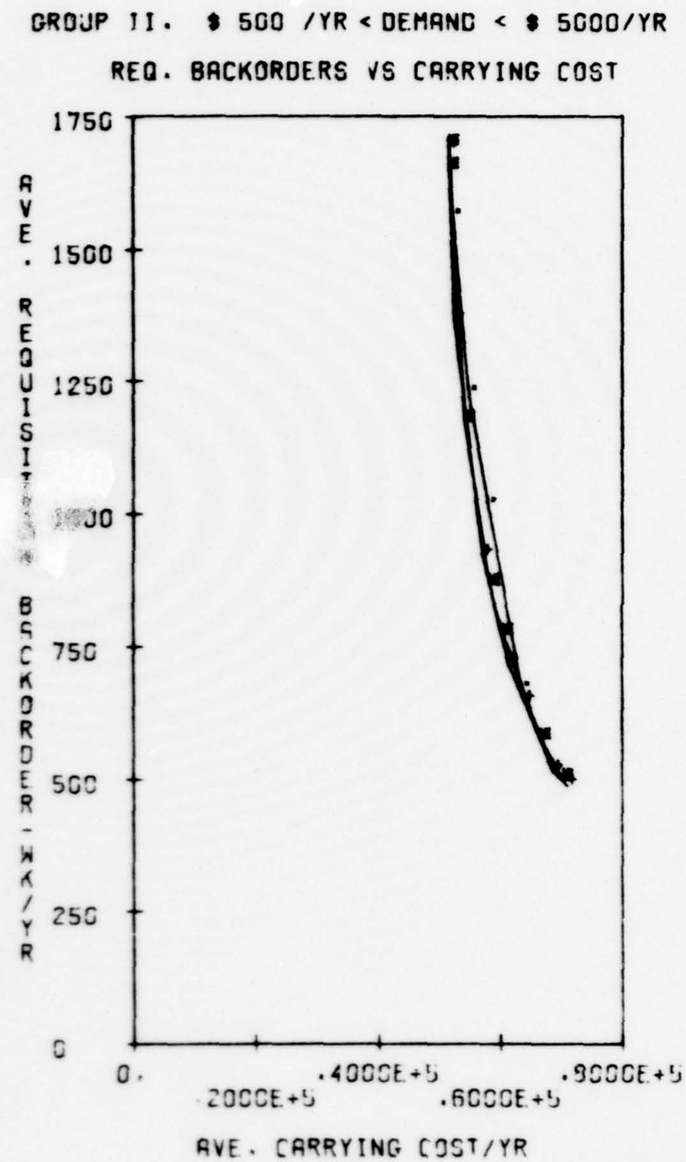
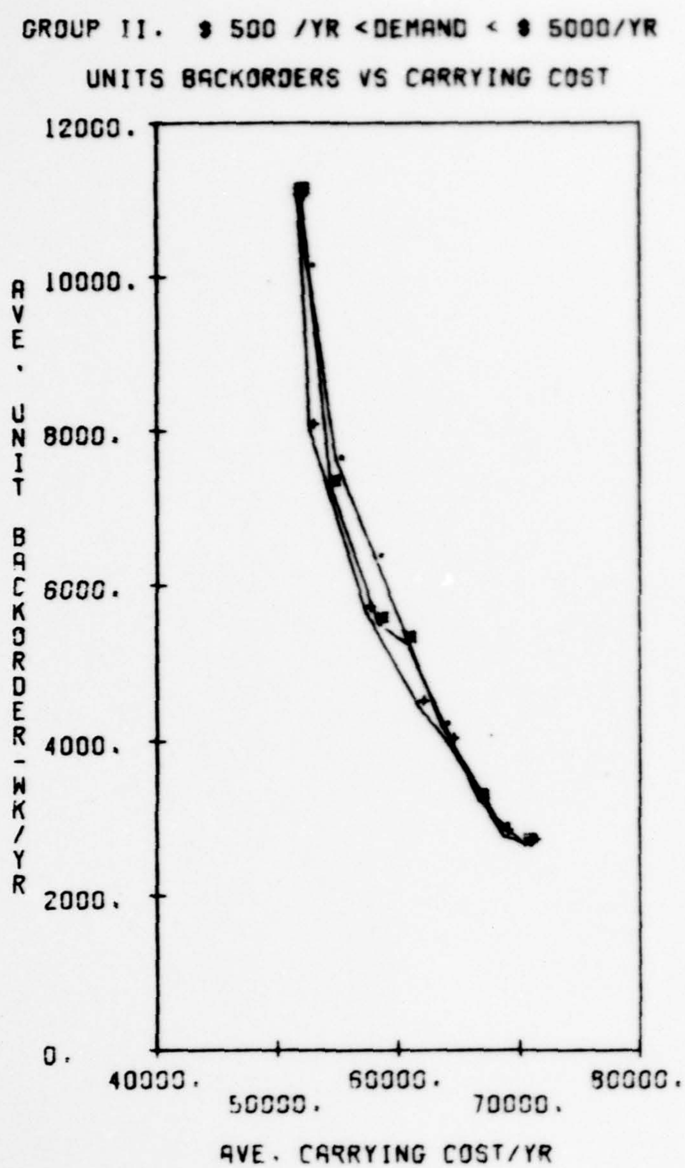


(b)

Figure V-6. Backorders verses Carrying Costs, MID Group.



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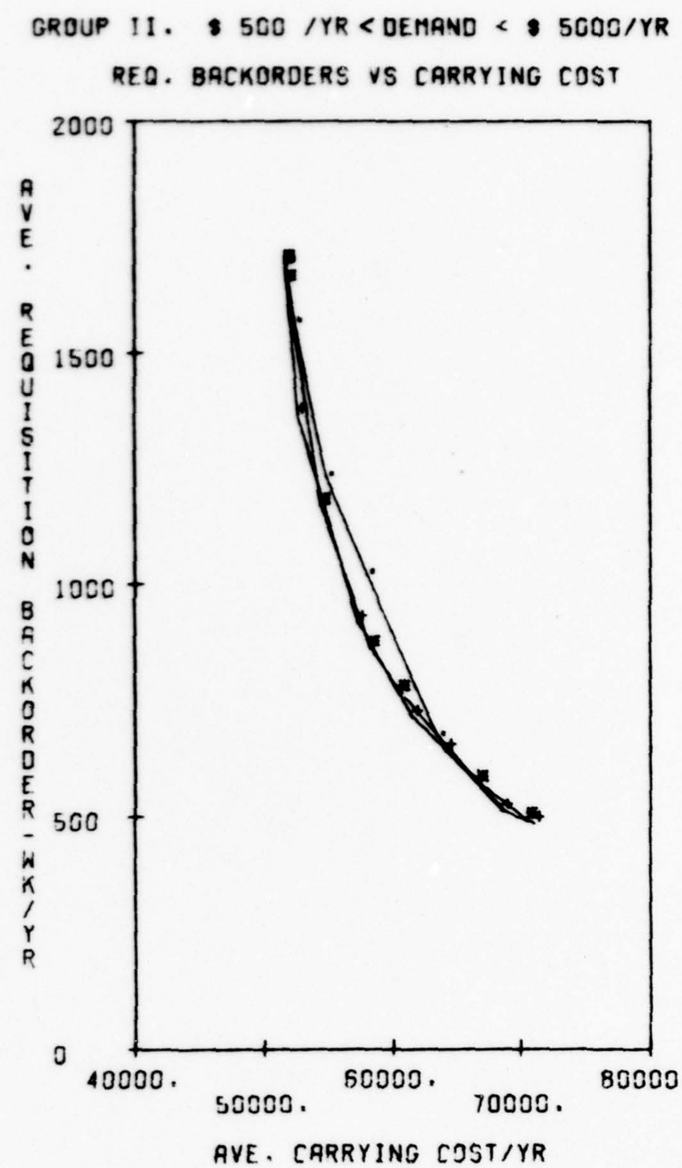


(b)

Figure V-7. Magnified Backorder/Carrying Cost Curve, MID Group.

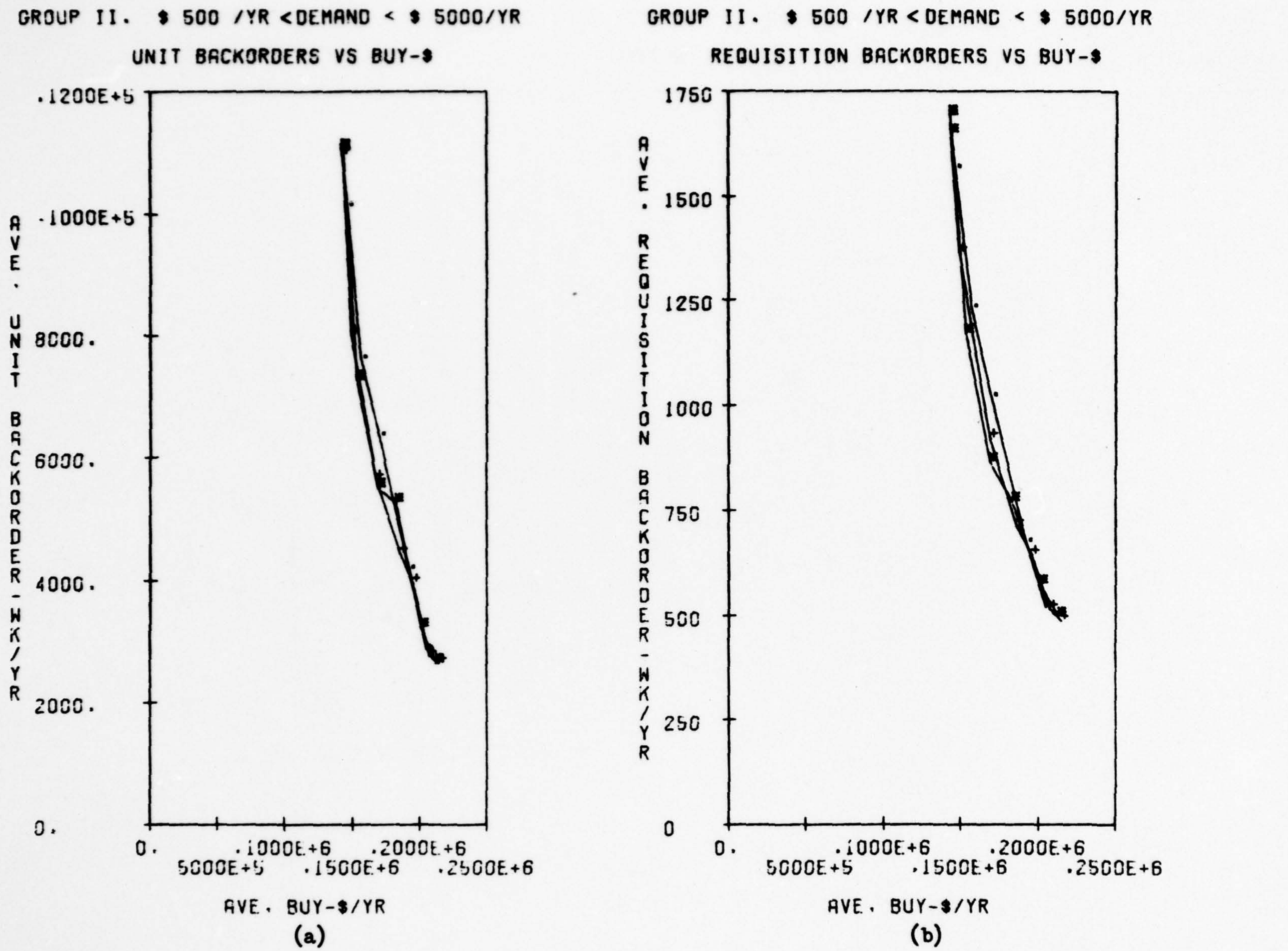


Figure V-8. Backorders verses Buy Dollars, MID Group.

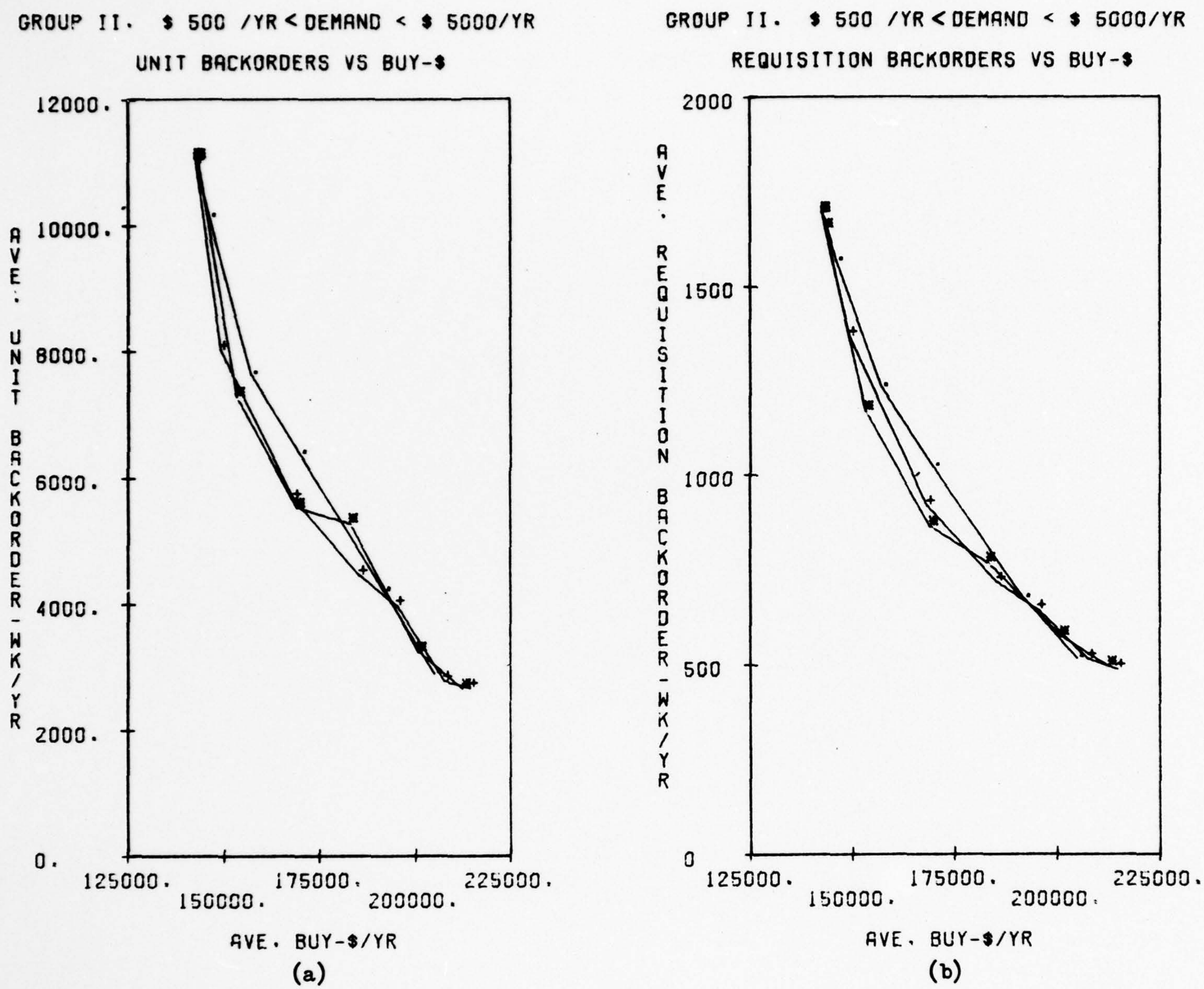


Figure V-9. Magnified Backorders/Buy Dollar Curve, MID Group.

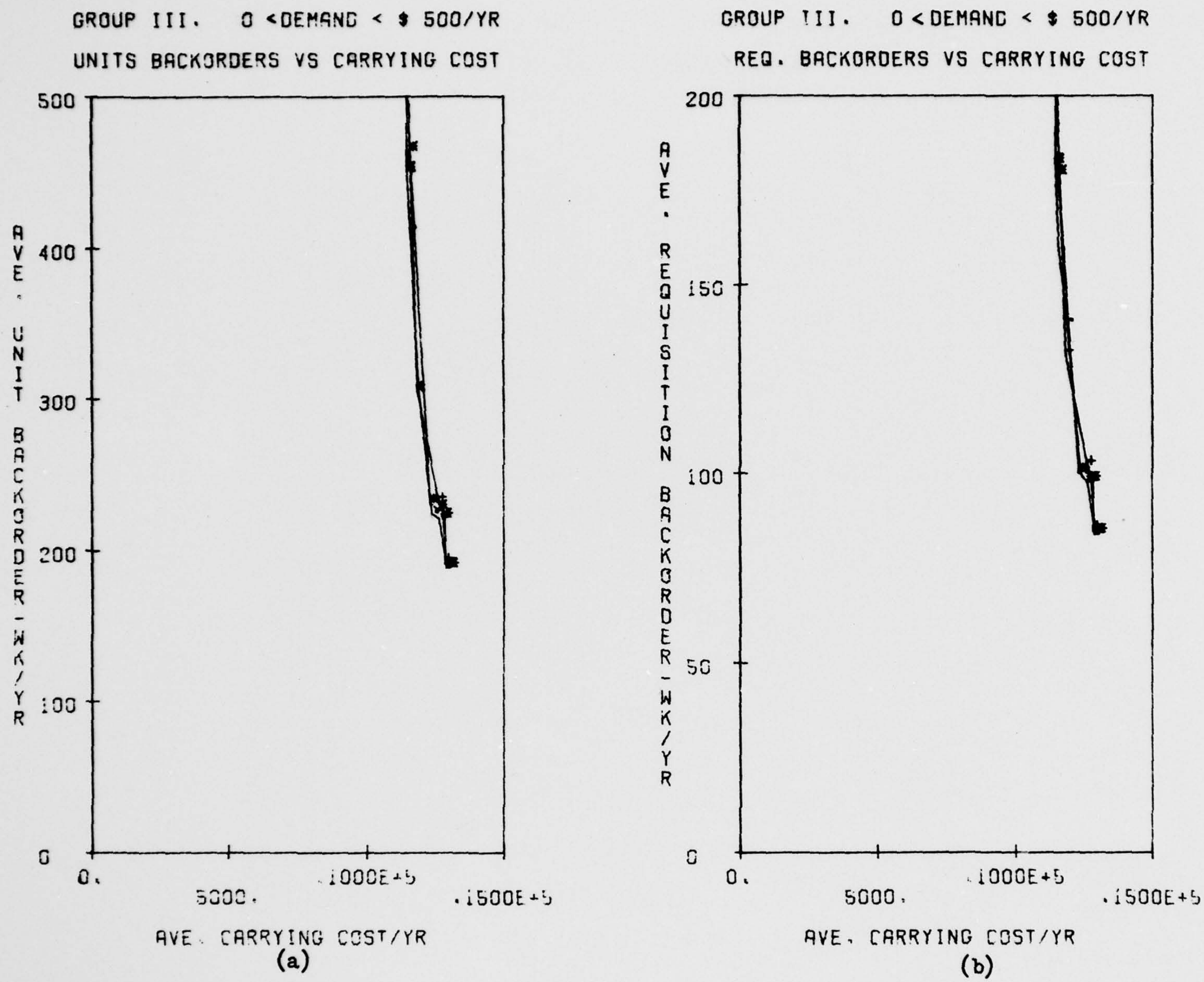
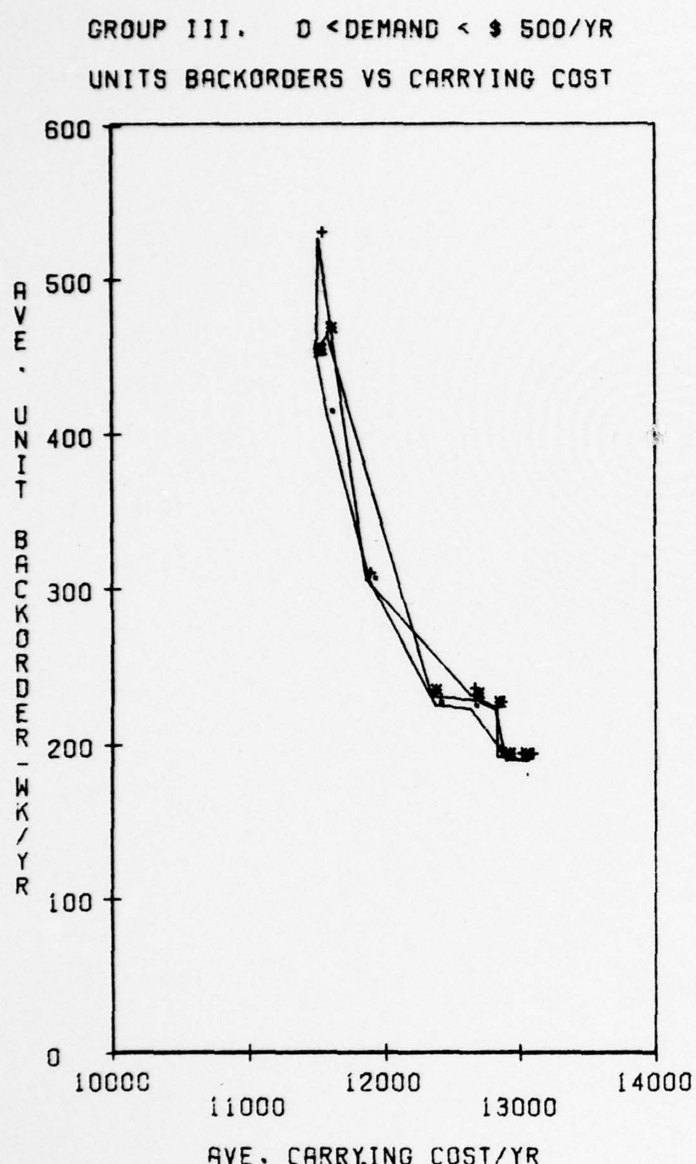


Figure V-11. Backorders verses Carrying Cost, LO Group.



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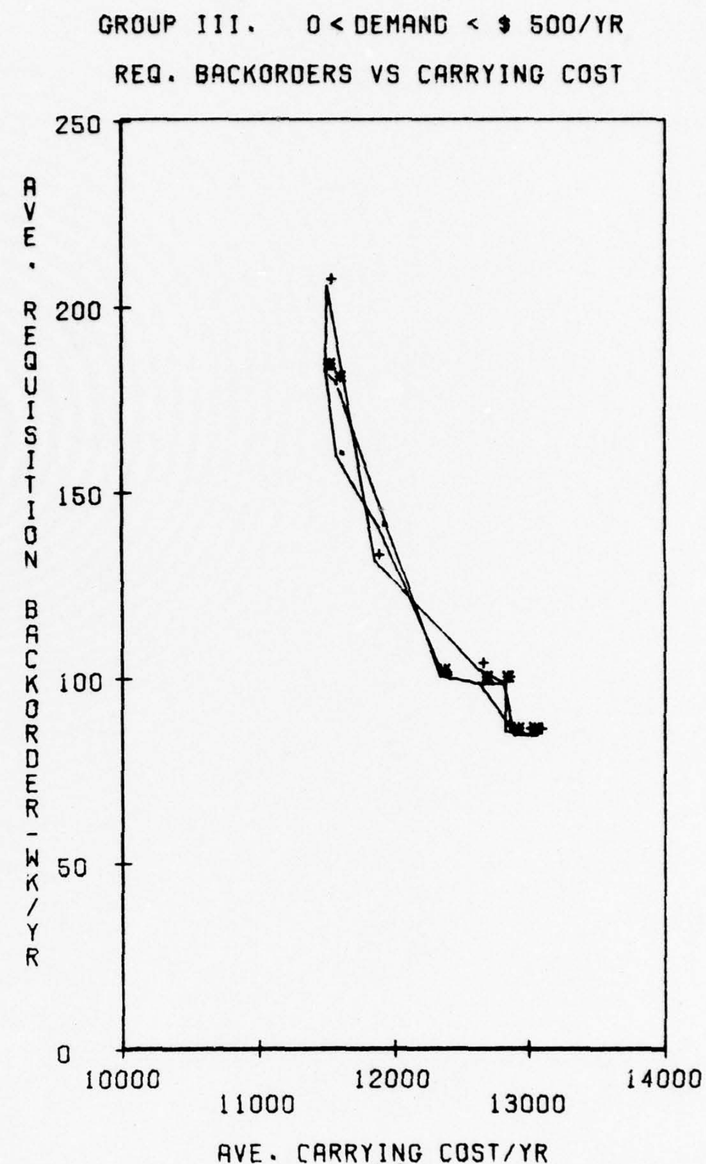
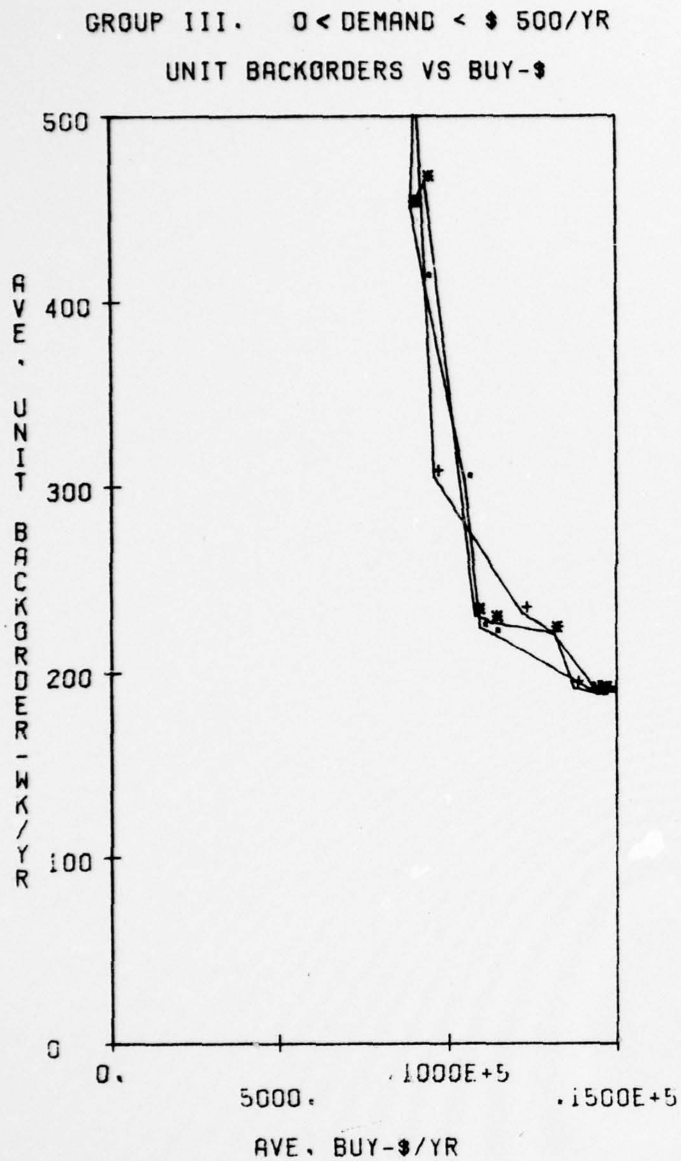


(b)

Figure V-12. Magnified Backorder, Carrying Cost Curve, LO Group.



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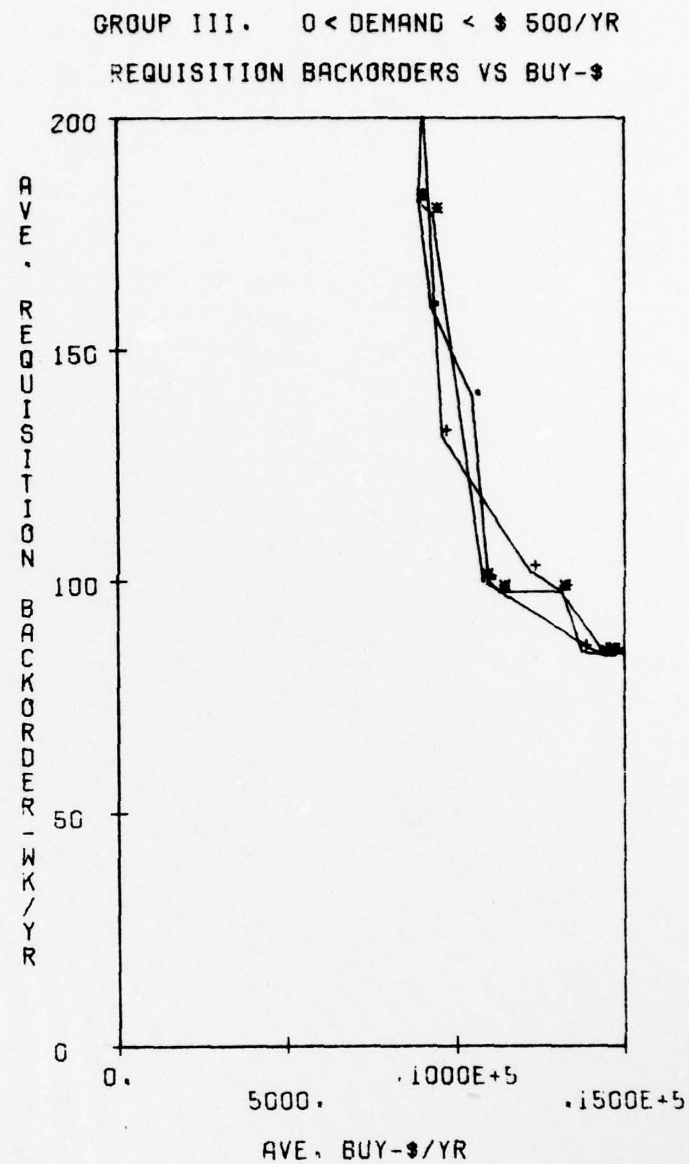

(b)

Figure V-13. Backorders verses Buy Dollars, LO Group.

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A STUDY OF THE COST-EFFECTIVENESS OF INVENTORY MANAGEMENT POLIC--ETC(U)

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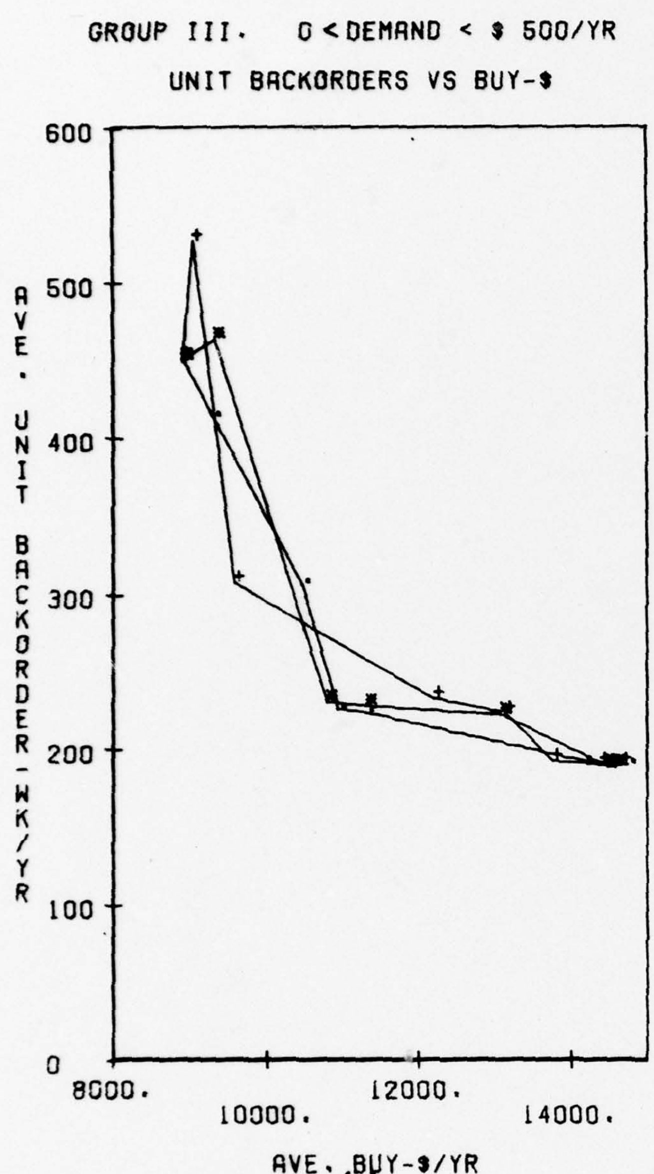

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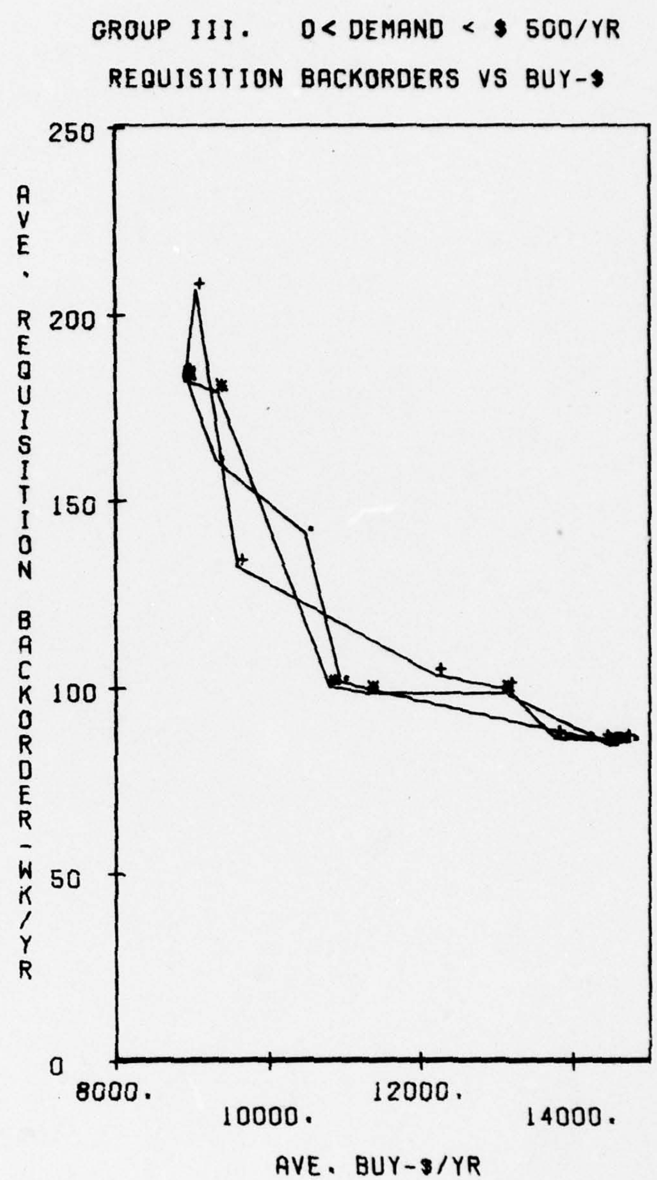


(b)

Figure V-14. Magnified Backorder/Buy Dollar Curve, LO Group.

few orders and demands during the three year simulation period. Due to the low activity of these items, the plots tend to be lumpy and irregular; it appears that much larger sample sizes will be required if smooth curves are desired. Based on the current results, no formula appears to be superior to the others in any of the plots. This latter result is not too surprising. We observed in Section II that items with low levels of demand tend to have average requisition sizes close to 1. When this happens, the alternate formulas become identical.

Samples by Unit Demand Class

Three additional samples were selected for simulation analysis. These latter samples were selected based upon the average unit demand rate during the FY71-FY72 period. In these samples, the following classes were used:

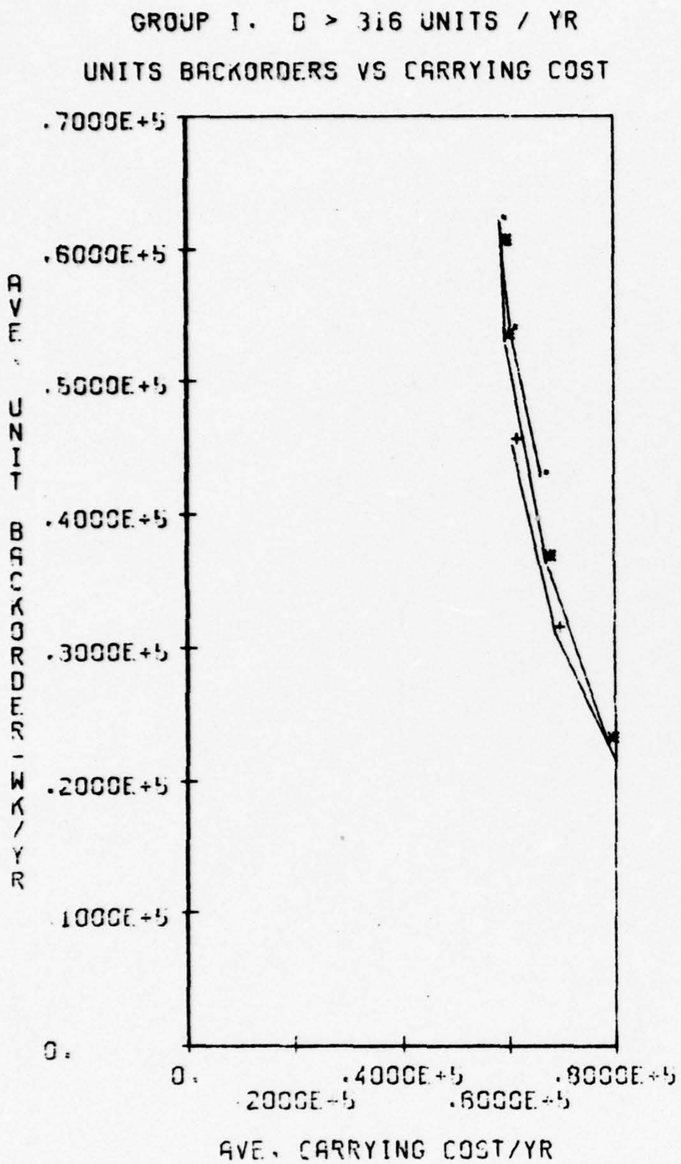
CLASS	Definition
I	Demand 316 units/year
II	Demand is between 31 and 316 units/year
III	Demand 31 units/year

Here, we are using Roman numerals as class identifiers to avoid confusion with the HI, MID, and LO Group samples discussed above.

As in the above studies, approximately 120 items were selected randomly from each of the classes I, II, and III. These items were selected for simulation analysis.

The results of these simluations are presented in Figures V-15 through V-20. In contrast to the above studies, only four runs were performed for each formula. These runs used λ values of \$10, \$31, \$100, and \$316, respectively.

In general, the cost-effectiveness curves for all of the formula sets are very similar. In Figure V-15, the Z=1 curve appears to clearly dominate the other two. However, there appears to be no clear dominance in any of the other plots.



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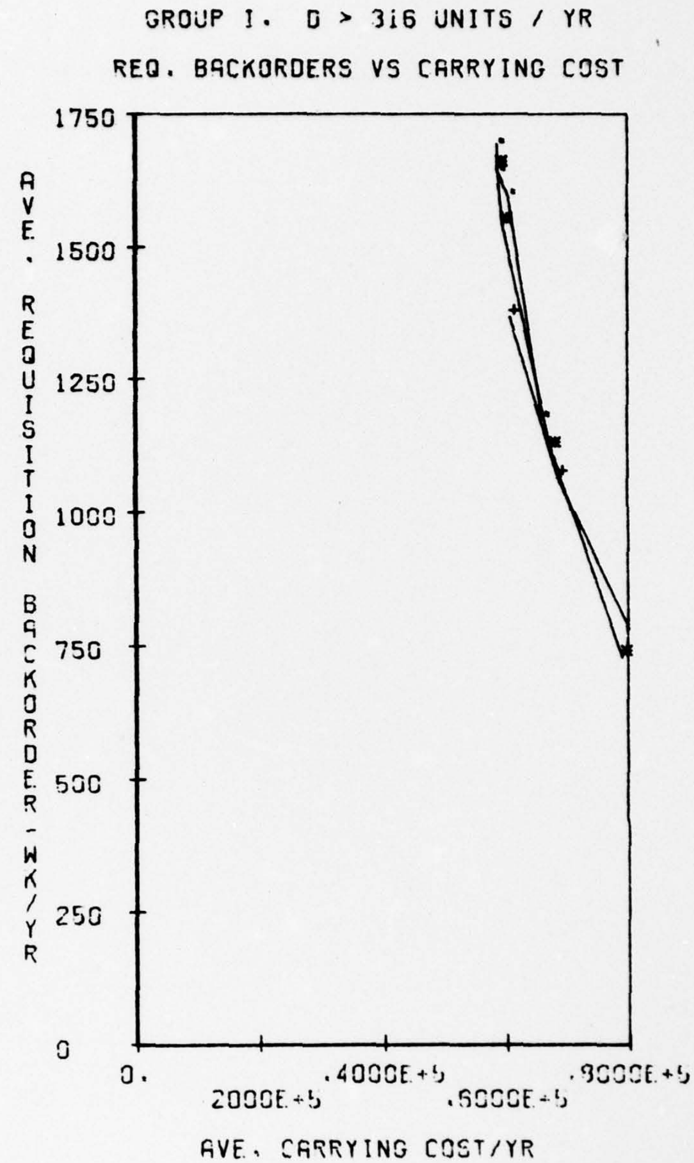


(b)

Figure V-15. Group I, Backorders verses Carrying Cost.

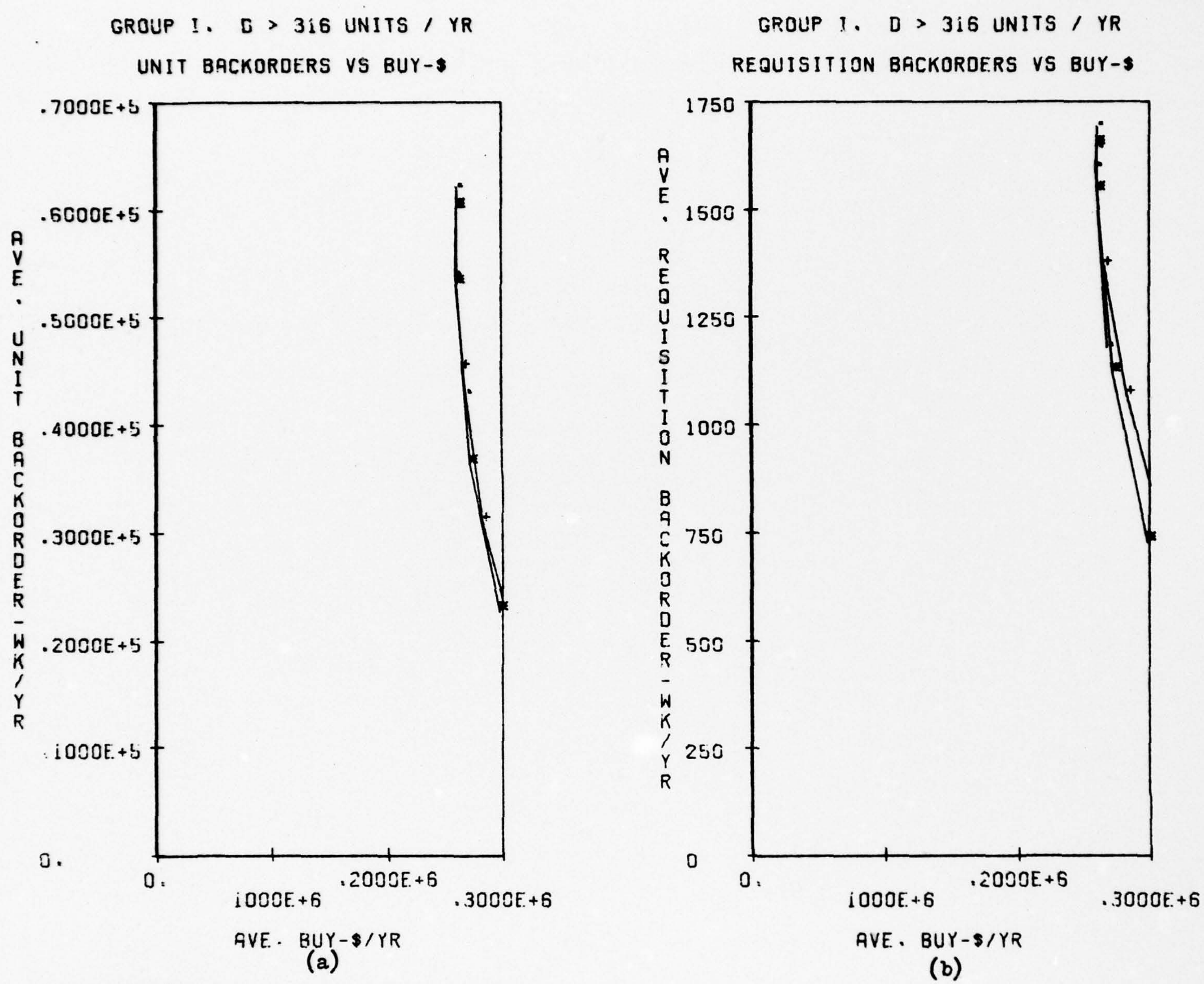
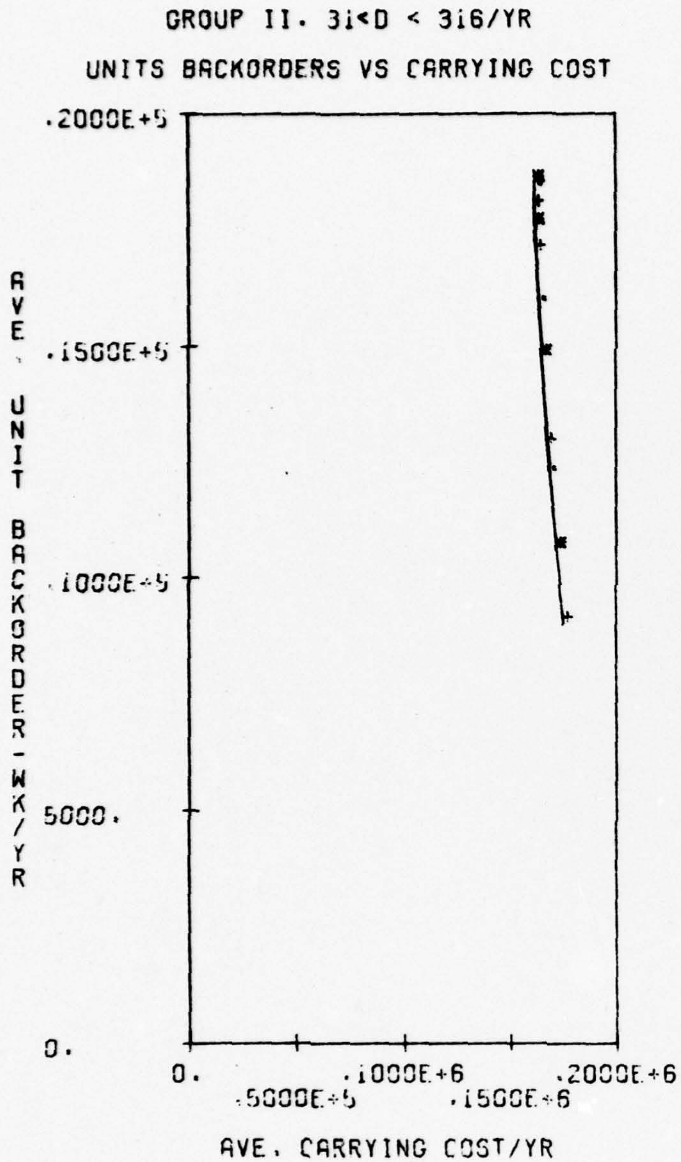


Figure V-16. Group I, Backorders verses Buy Dollars.



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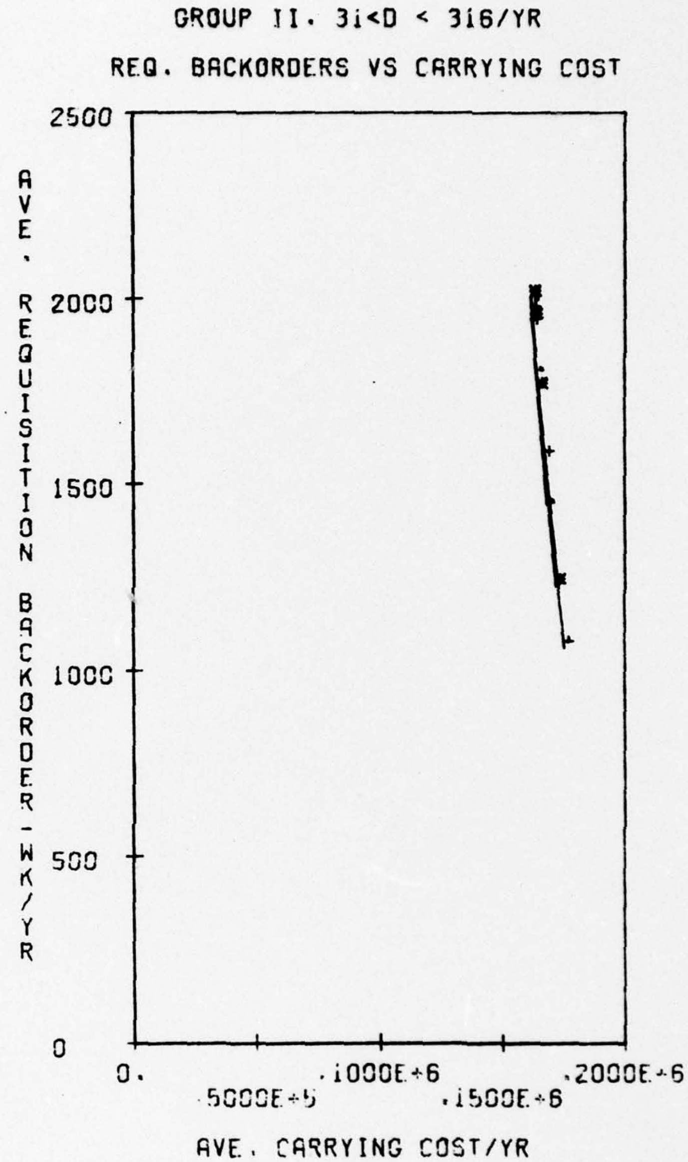


(b)

Figure V-17. Group II, Backorders verses Carrying Cost.

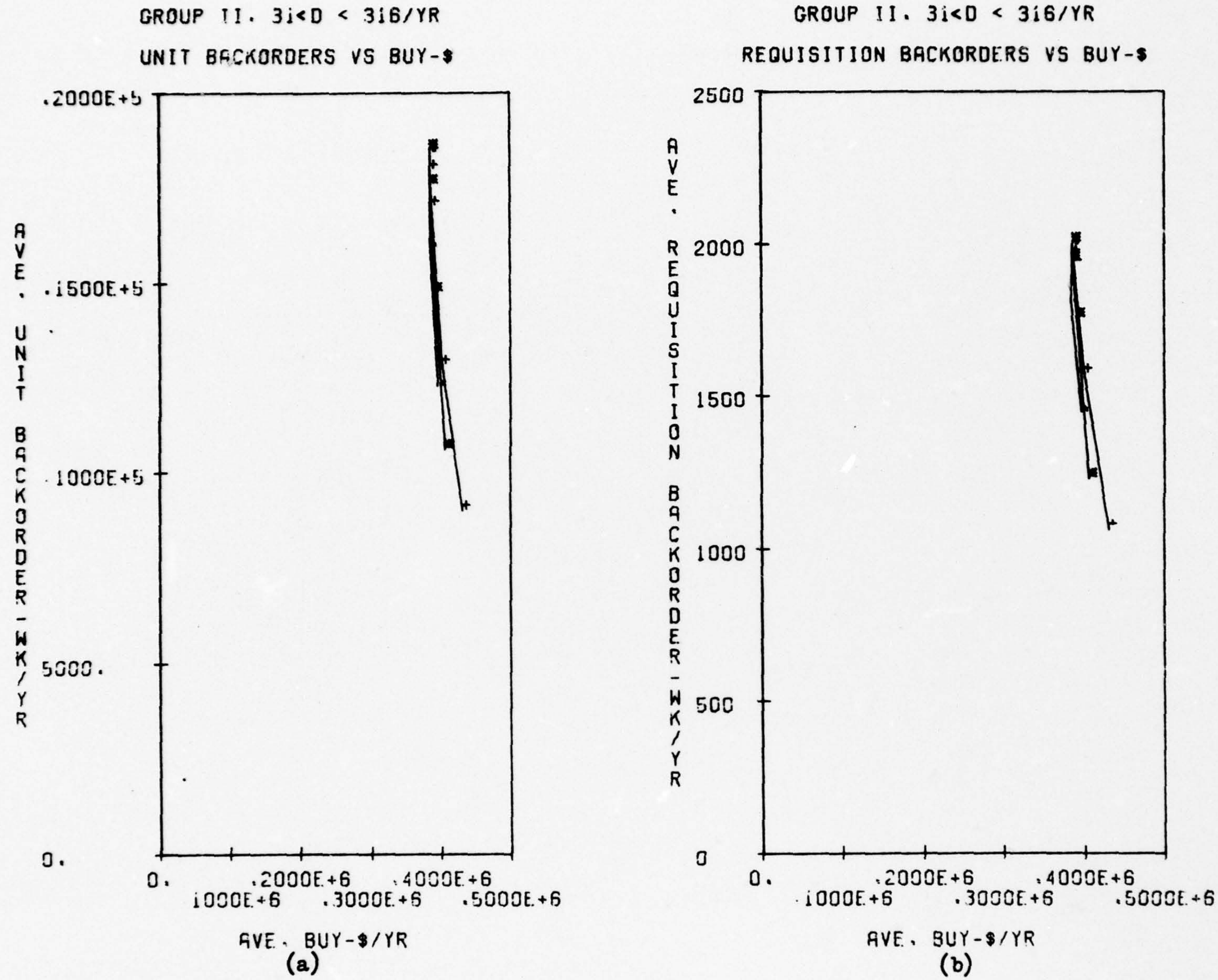
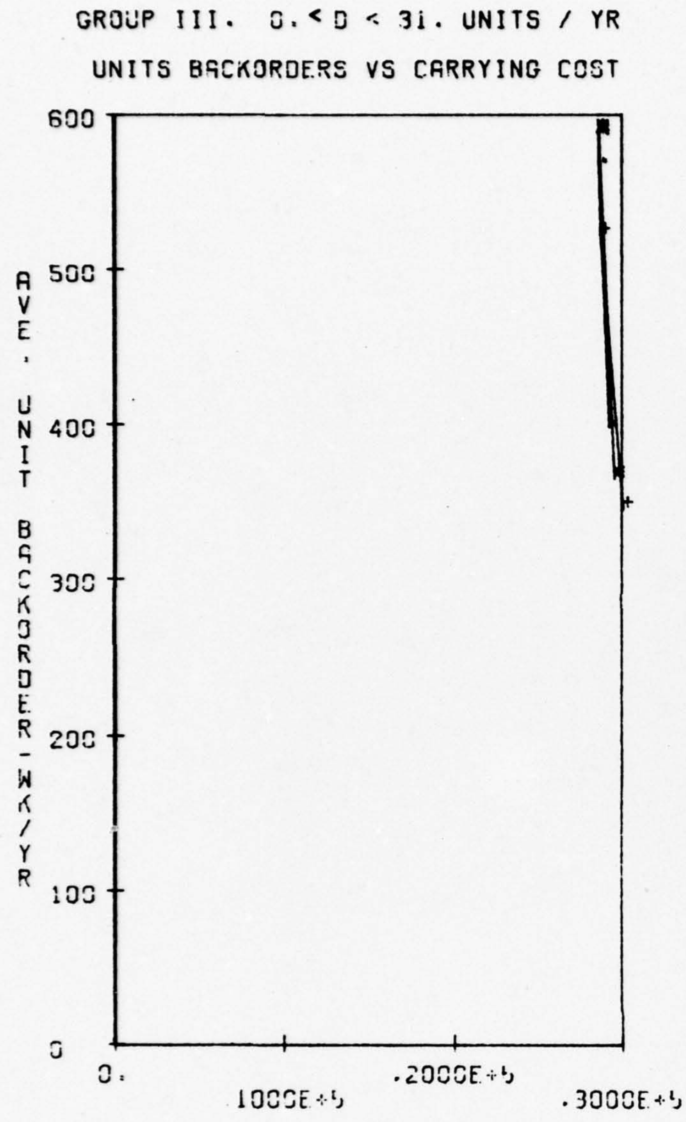


Figure V-18, Group II, Backorders verses Buy Dollars.



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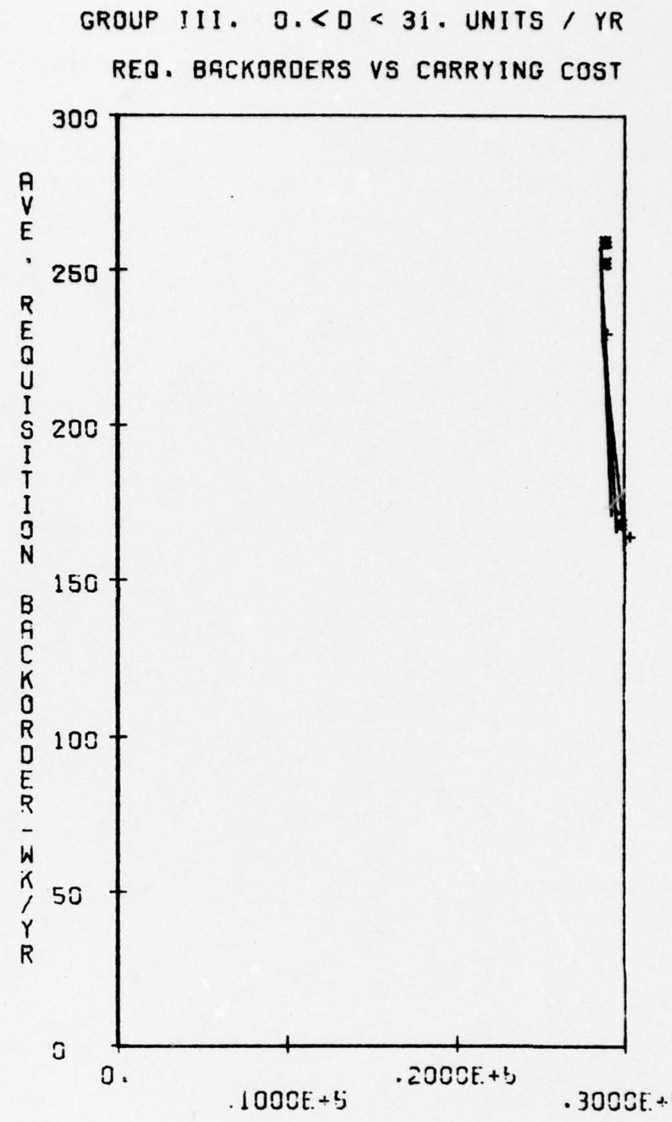
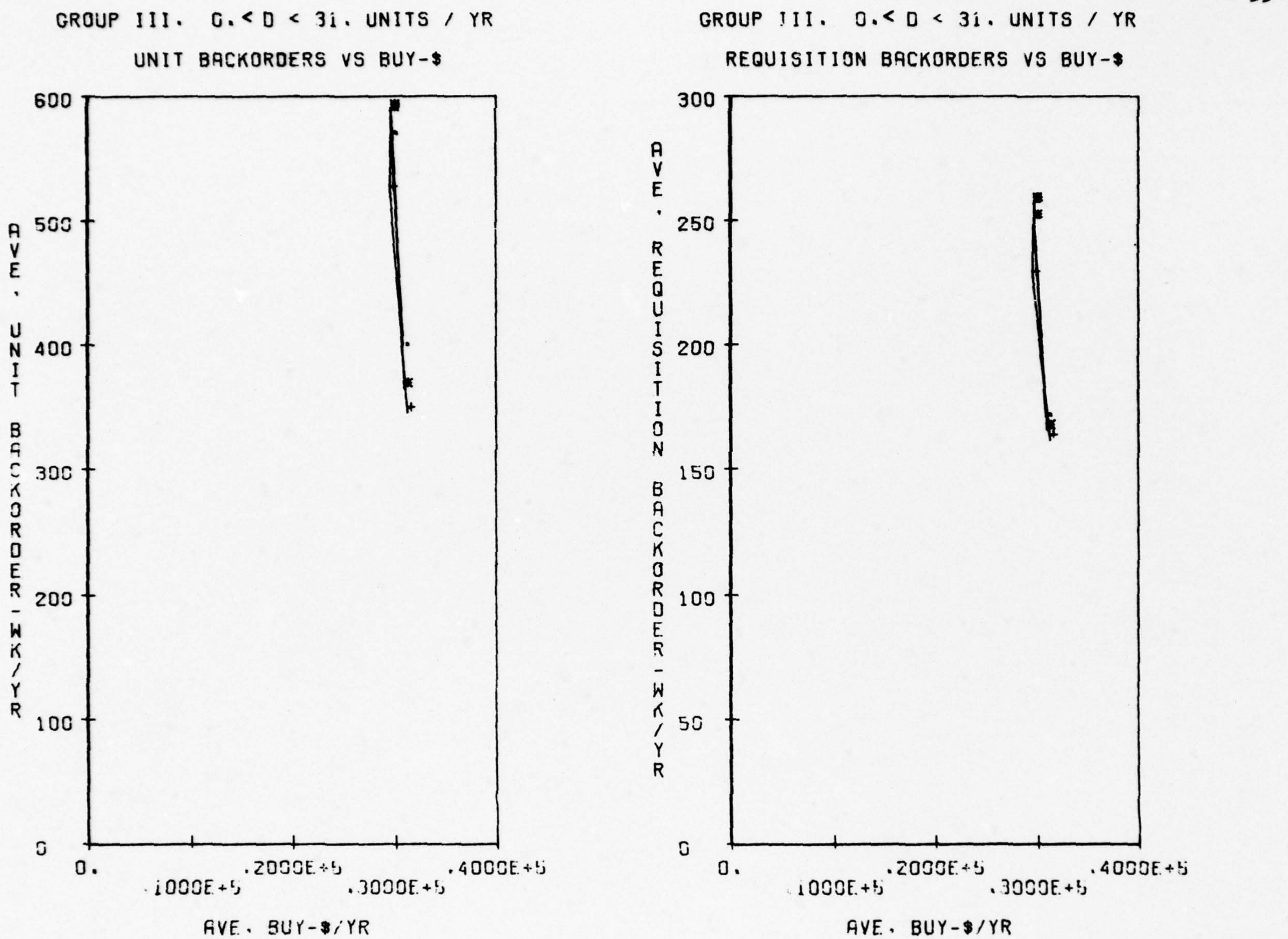


(b)

Figure V-19. Group III, Backorders verses Carrying Cost.



(a) (b)

Figure V-20. Group III, Backorders verses Buy Dollars.

STATISTICAL ANALYSIS

The above results compare the relative cost-effectiveness of the alternate PT-formulas in managing each item sample. However, an important question remains--namely, with what confidence would we expect similar results if we had simulated every EOQ item, rather than just a sample. Although we exercised great care in selecting and editing the item samples, there is still a chance that the samples are not representative of the total EOQ population.

Several analysis of variance studies were conducted to measure the magnitude and the statistical significance of differences observed in the cost-effectiveness curves for each formula set. In these analyses, we divided each item sample into two groups of 50 items each. We then hypothesized that the number of unit backorders in a given period in a given simulation run was a function of several effects: (1) a formula effect, (2) a time period effect, and (3) a group effect. We wished to measure the magnitude and statistical significance of each of these effects when the same amount of money is invested in inventories in each case. This led to an analysis of the following statistical model and its variations.

$$E_{ijkt} = A + F_i + T_j + G_k + B \cdot I_{ijkt} + C \cdot I^2_{ijkt} + e_{ijkt}$$

where i = formula set index

j = time period index

k = item group index

t = implied shortage cost (λ) index

The symbol E_{ijkt} denotes the average annual backorders observed in the j^{th} simulated year when item group k was simulated using formula set i and using the t^{th} value of the implied shortage cost parameter λ. The symbol A denotes the average aggregate backorders observed for an item group, while F_i, T_j, and G_k denote the amount of variation from this average associated with the formula set, time period, and group effects, respectively. The symbol I_{ijkt} denotes the the average level of inventory for group K observed in the j^{th} time period when formula set i was simulated. Hence, the parameters B and C measure the average linear and quadratic effects of the inventory investment upon unit backorders. Finally, the symbol "e" denotes the amount of variation in backorders E that is not explained by the above model.

Unfortunately, our results were quite disappointing. Although the above model has significant initiative appeal, it did not appear to fit our data very well. In particular, it appeared that the group differences have far more impact upon backorders than reflected in the linear model shown above. Unfortunately, we were unable to discover a better analytical model.

Based upon the above statistical model, group and time period effects have a significant impact upon unit backorders and upon requisition backorders; however, there appeared to be no statistically significant difference among the formulas that could be measured in the face of variability introduced by group and time period differences.

To summarize, we were unable to construct a statistical model that provided a good description of the observed results. For the best (but imperfect) model that we found, there was no statistically significant differences among the formula sets. Hence, it appears that additional research is required to make statistically significant estimates of the differences among these formulas.

Section VI. Summary and Conclusions

The objective of this study has been to evaluate the relative cost-effectiveness of the three versions of the Presutti-Trepp formulas defined in Appendix A. These formula sets are identical except for the value of the factor Z used in the safety level computation. In formula set II, Z=1. This formula set contains the PT-formula for minimizing expected units backordered. In formula set III, Z=R= average requisition size. It may be argued analytically that this formula set tends to minimize expected requisitions short. Finally, for formula set I (the current D062 computation formulas), $Z=\sqrt{R}$.

In Section II, we reviewed the statistical characteristics of Sacramento EOQ items. This data shows that most EOQ items have very low demand rates. In fact, out of 78,436 items, 66,229 had no demands at all during the period July-September 1976. Overall, more than 96% of the items had demand of less than 31 units during this period. For those items that had at least one demand, over 90% of them still had demands of less than 31 units. Hence, most EOQ items have very low unit demand rates.

The distribution of average requisition size by unit demand class is presented in Figure II-2. As expected,

we found that average requisition size increases as unit demand increases. For example, the median requisition size for items with 11-32 demands per quarter equals 2.5, while items with 317-1000 units per quarter have a median size of 21 units per requisition. However, since most EOQ items have very low demand rates, we would expect the average requisition size for most items to be small, e.g. 3 or less.

In Section III, we studied the computation characteristics of the PT-formulas. Several curves were presented in this section illustrating the sensitivity of the formulas to changes in input parameters.

Finally, Section IV presented a general outline of a simulation model to evaluate the relative cost-effectiveness of the alternate PT-formulas, and Section V presented results of simulation runs using this model.

All the simulation runs were based on actual demand data for Sacramento ALC items for the FY 71 through FY 76. Two categories of runs were conducted. The first category of runs were based on item samples stratified by the average annual dollar value of demands in the FY 71 to FY 72 period. Three samples of items were then developed by random sampling within high, moderate, and low activity classes. The HI class consisted of items with demands in excess of $5,000/year during this period, and the LO

class consisted of items with average demands of less than $500/year during this period. The MID class consisted of items between these extremes. The second category of simulation runs was based upon item samples stratified by the average annual unit demand rates in the FY 71 to FY 72 period. In this latter case, class I consisted of items with more than 316 units/year, class III consisted of items with demands of less than 31 units/year, and class II consisted of items between these extremes.

For each class, we obtained 100 items by random sampling from the Sacramento MOO Data Bank. Then, each class was simulated using each of the three formula sets and a variety of values for the shortage factor. Figures V-2 through V-20 present the results of these runs.

In general, we observed very little difference among the cost-effectiveness curves for each of the three formula sets. In particular, there appears to be no clear differences among items in the low and moderate activity classes, i.e., in the classes LO, MID, II, or III. Perhaps this is to be expected, since items with low activity tend to have low average requisition sizes. As the average requisition size approaches one, the three formula sets become identical.

For the high activity classes, i.e., class HI and I, clear differences were observed among the alternate formulas. For both the HI class and class I, the Z=1 formula appeared more cost-effective than the other two formulas in minimizing unit backorders; and the $Z=\sqrt{R}$ formula appeared better than the Z=R formula in this respect. However, if we were interested in minimizing requisitions-backordered, there were no clear differences among the curves.

In summary, for the high activity classes, the Z=1 formula appeared superior in minimizing unit backorders, and it appeared about equal with the other formulas in minimizing requisitions short.

The above comments are based upon simulation analysis of random samples of 100 items in each class. However, an important question remains -- namely, with what confidence would we expect similar results if we had simulated every EOQ item, rather than just a sample. Although great care was exercized in selecting and editing the item samples, there is still a chance that the item samples are not representative of the total EOQ population.

To answer this last question, several statistical analyses were conducted to measure the magnitude and statistical significance of differences observed in the

cost-effectiveness curves for each formula set. Unfortunately, our results were quite disappointing. We were unable to construct a statistical model that provided a good description of our observed results. For the best (but unsatisfactory) model that we found, there was no statistically significant differences among the formula sets. For this analysis model, it appears that variability due to group and time period effects drowned out any consistent differences that may exist among the alternate formulas.

In summary, in our simulation runs the B=1 formula appeared superior to the other formulas for the high activity item classes. On the other hand, there were no clear differences among the alternate formulas for the low and moderate activity groups. In addition, the differences that we did observe did not appear to be statistically significant. Hence, it appears that additional research is required to make precise estimates of the differences among these formulas. To provide these estimates, analyses of data samples from the Oklahoma City and Warner Robins ALCs are currently in progress.

References

1. Presutti, Victor J. and Richard C. Trepp, "More Ado About Economic Order Quantities", Naval Research Logistics Quarterly, v 17, n 2, June 1970, pp. 243-251.

2. Demmy, W. Steven, The Inventory System Simulator, Volume I: The Model, Technical Report 77-2, Decision Systems, 3575 Charlene Drive, Dayton, Ohio 45432.

3. Demmy, W. Steven, The Inventory System Simulator, Volume II: Programs, Technical Report 77-2, Decision Systems, 3575 Charlene Drive, Dayton, Ohio 45432.

Appendix A

Alternate Versions of the Presutti-Trepp Formulas

The formulas to be evaluated are as follows:

Formula Set I. (Current D062 Reorder Level and Buy Quantity Rules)

A. Monthly Demand Rate (MDR) =

$$(1)\quad \left. \sum_{n=1}^{N} \left[(\text{Gross Demands}_n) - (\text{serviceable returns}_n) \right] \right/ N$$

where N = Base period quarters

GROSS DEMANDS = TRANSFER + SALES DEMANDS
SERVICEABLE RETURNS = TRANSFER + SALES SERVICEABLE RETURNS
BASE PERIOD = 2 YEARS.

B. Reorder Level (ROL) = Leadtime demands + safety level

(2) Leadtime demands = Leadtime months X MDR

(3) Safety Level = K X σ

where

$$(4)\quad \sigma = 0.5945 \text{ X } MAD_Q \text{ X}(0.82375 + 0.42625 \text{ X LEADTIME (LT) MOS})$$

$$(5)\quad MAD_Q = \left. \sum_{n=1}^{N} \left[\left| \text{Actual Quarterly Demand}_n - 3 * MDR \right| \right] \right/ N$$

where N = base period quarters

$$(6)\quad K = 0.707 \text{ X LN} \left[\frac{\begin{pmatrix}\text{Implied}\\ \text{Shortage}\\ \text{Factor}\end{pmatrix}}{2 \cdot \begin{pmatrix}\text{Holding}\\ \text{Cost}\end{pmatrix} \cdot \begin{pmatrix}\text{Unit}\\ \text{Cost}\end{pmatrix}} \cdot \frac{1}{Z} \cdot \frac{\sigma \cdot (1 - \text{EXP}(-\sqrt{2}\, EOQ/\sigma))}{\sqrt{2}\, EOQ} \right]$$

where $Z = \sqrt{\text{Average Requisition Size}}$, and

$$(7)\quad \text{Average Requisition Size} = \frac{\sum_{n=1}^{N} \text{GROSS DEMANDS}_n}{\sum_{n=1}^{N} \text{FREQUENCIES}_n}$$

C. MINIMUM & MAXIMUM CONSTRAINTS ON SAFETY LEVEL
MINIMUM = ZERO
(8) MAXIMUM = LESSER OF 3σ OR LEADTIME DEMANDS

D. ECONOMIC ORDER QUANTITY (EOQ)

(9) $$\text{EOQ} = \sqrt{\frac{2 \text{ X MDR X } 12 \text{ X ORDERING COST}}{\text{HOLDING COST X UNIT COST}}}$$

WHERE HOLDING COST AND ORDERING COSTS ARE INPUT CONSTANTS.

E. CONSTRAINTS ON EOQ

(10) MINIMUM EOQ = 6 X MDR
MAXIMUM EOQ = 36 X MDR

F. BUY QUANTITY
BUY QUANTITY = ROL DEFICIENCY + EOQ

ROL DEFICIENCY = ROL - ASSET POSITION

WHERE

ASSET POSITION = ON HAND ASSETS + DUE-INS - BACKORDERS

Formula Set II. This formula set is identical to SET I, with the exception that Z=1 in equation (6).

Formula Set III. This formula set is identical to Set I, with the exception that Z=Average Requisition Size in equation (6).

Appendix B

Record Layout for the EOQ Data Bank History File

INPUT /OUTPUT RECORD

SYSTEM NUMBER

PAGE 1 OF 1 PAGES

TITLE
EOQ DATA BANK HISTORY FILE (Old Format)

INPUT / OUTPUT	FILE NUMBER	RUN NUMBER	SEQUENCE OF FILE

DATA RECORD LENGTH
124

TAPE RECORD LENGTH/ BLOCKING FACTOR
SSF

REMARKS

This format applies to History Tapes created prior to Jun 1976

NO.	DATA ELEMENT NAME	POSITION OR BLOCK	STRUCTURE LENGTH	STRUCTURE CHARAC
1	ALC	1	2	AN
2	NSN	3	15	AN
3	UNIT COST	18	9	N
4	SPECIAL CODE	27	1	AN
5	LEAD TIMES	28		
	ADMINISTRATIVE	28	2	N
	PRODUCTION	30	2	N
6	TRANSFER DEMANDS	32		
	1ST QTR OF FY (1 JUL - 30 SEP)	32	7	N
	2ND - 4TH QTRS OF FY	39	21	N
7	NON-RECURRING DEMANDS	60		
	1ST QTR OF FY (1 JUL - 30 SEP)	60	7	N
	2ND - 4TH QTRS OF FY	67	21	N
8	TRANSFER & SALES SERVICEABLE RETURNS	88		
	1ST QTR OF FY (1 JUL - 30 SEP)	88	7	N
	2ND - 4TH QTRS OF FY	95	21	N
9	FREQUENCY OF DEMANDS	116		
	PAST 6 MONTHS	116	4	N
	CURRENT 6 MONTHS	120	4	N

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REPORT DOCUMENTATION PAGE

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5. TYPE OF REPORT & PERIOD COVERED: Interim
6. PERFORMING ORG. REPORT NUMBER: RM 77-01
7. AUTHOR(s): W. Steven Demmy

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19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

study, cost-effectiveness, inventory management, EOQ, requisition-size, Presutti-Trepp, INSSIM model

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report presents the results of a study of the relative cost-effectiveness of three variations of EOQ inventory management calculations in the context of Air Force depot supply systems. These formulas differ in the manner in which average requisition sizes are utilized in inventory levels calculations. The report contains five sections. Section I provides background for the study. Section II presents aggregate statistical characteristics for EOQ items managed by the Sacramento Air Logistics Center (SMALC). Section III presents the results of analytical sensitivity analyses

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20. Abstract

of the alternate formulas. Section IV discusses the major features of a retrospective inventory simulation model for evaluating the alternate policies, and Section V presents the results of simulation analyses based on this model and upon historical demand data from SMALC.